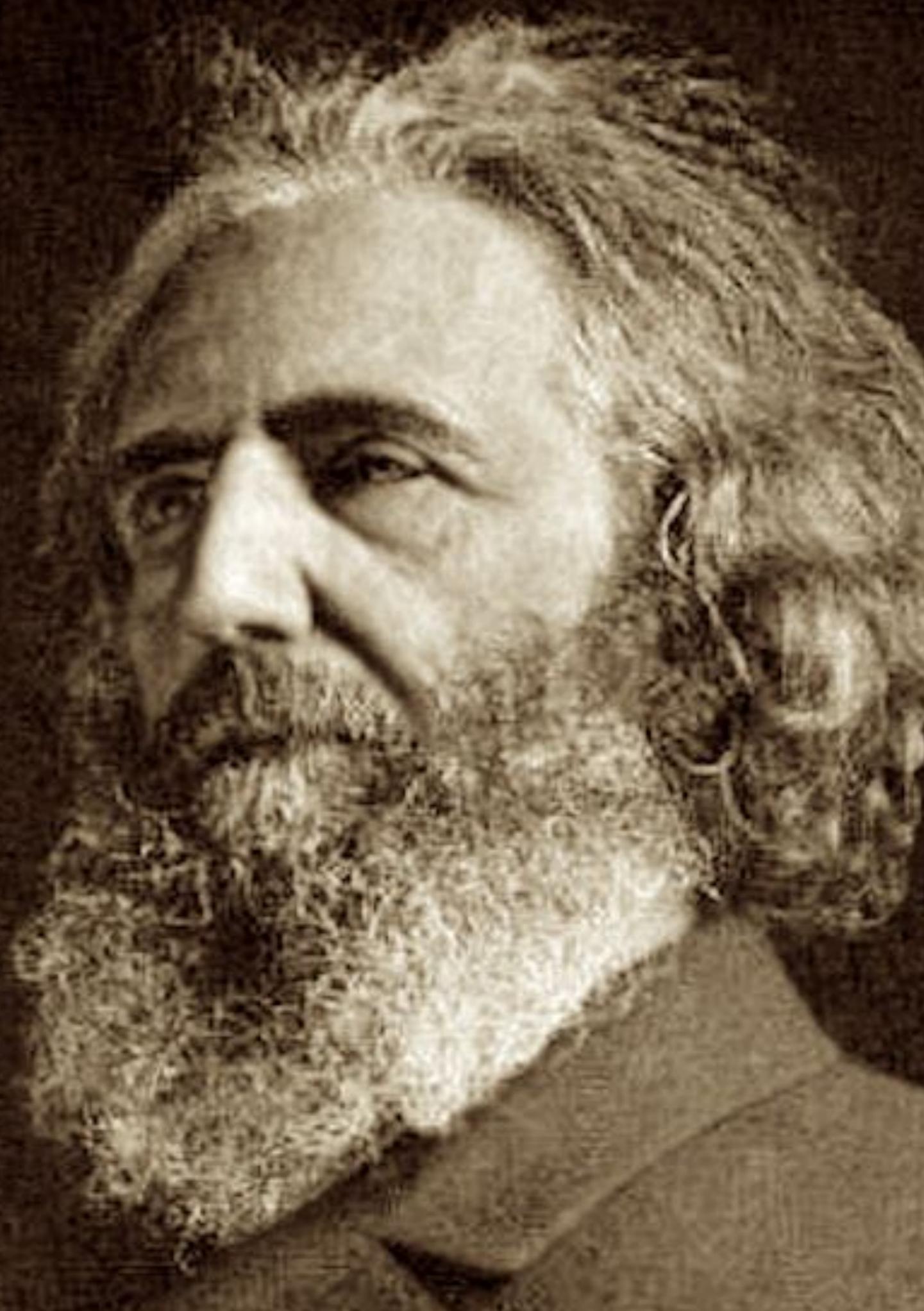


Points

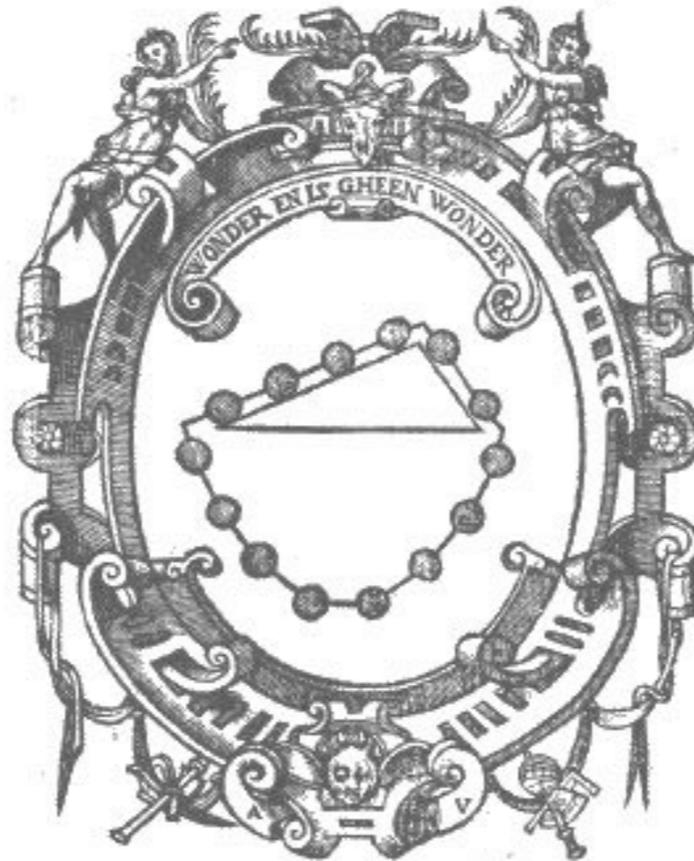
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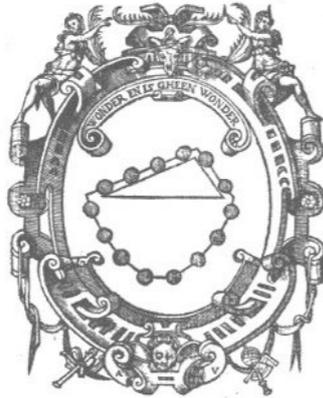
Points

Jan Koenderink



DE CLOOTCRANS PRESS, MMXVII

Front cover: A portrait of Franz Brentano.



[De Cloutcrans Press](#)

Utrecht The Netherlands

KoenderinkJan@gmail.com

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pax / jan koenderink

First edition, 2017

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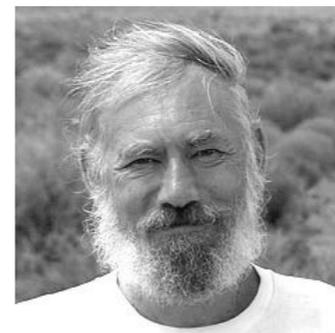
Foreword

This short [E-Book](#) is made up of the slides of a presentation I gave at the Justus Liebig Universität, Giessen (in Hessen, Germany) in 2016. I “recycled” the talk at Vielsalm (Belgian Ardennes) in 2017, at a retraite of Gestalt psychology of the KU Leuven.

Of course, the slides alone cannot convey what I presented. So I interspersed some pages with additional comments. When I find the time I will make up for this by means of Internet references. However, this may take another year or so.

I’m a retired professor, time has since long lost its meaning to me.

Utrecht, april 9, 2017 — Jan Koenderink



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POINTS

In this chapter I explain a bit what the talk was supposed to be about. For most of the audience the title made no sense and — especially in the initial quarter of an hour (I talked for an hour and a half) — they felt it was a waste of their time. When I came to talk on “perceptive fields” as complementary to “receptive fields” this somewhat rekindled their interest. A typical psychology audience feels best when lectured on (neuro-)physiology nowadays. My attempt to get them interested in their proper field only partly succeeded, I think.

Franz Brentano’ Plerosis

The original title of the talk was FRANZ BRENTANO’S PLEROSIS: POINTS, LINES & EDGES. What is “plerosis” anyway? It was Brentano’s solution to the structure of “points”.

Points have been enigmatic objects since the dawn of time, the problems deriving from the structure of the continuum. Brentano was aware of the Greek philosophers, although he drew mainly on Aristotle. In his time mathematics had made great strides in analysis and one needed to come to terms with the nature of the “real numbers” and the “infinitesimals”. This involved a modern interpretation of human thought on these matters since the presocratics. Brentano’s ideas are to a large extent reactions to what he considered unfortunate aberrations such as the notion of “Dedekind’s cut”.

Is the talk about history then? Not really, the topic is as lively as ever! However, it has lost much of its innocence. We have a somewhat broader perspective than people at Brentano’s time could have. This by no means implies that all our riddles have been solved though!

The talk centered on such riddles in the context of (mainly) experimental phenomenology, that is really — or should be — psychology and neuroscience, or brain theory.

In order to get on course I needed to cover a bit of the history of human thoughts on the nature of the continuum. This involves some formal mathematics, albeit of the simplest kind. However, these simple things reveal or hide (up to you!) major conceptual issues.

The talk certainly succeeded in capturing the attention of the audience and was generally received as entertaining. Did I actually “reach” the audience? I guess mostly not. Several people told me that there *are* points in the blue sky, they apparently believe in Newton’s absolute space. Someone warned the youngsters that all this was nice and interesting, but of no importance to experimental psychology. Maybe he was right.

At the actual talks the audiences had the advantage of listening to me talking circles about the slides they are presented with, something that you, as mere reader, have to miss. I often hear myself relate odd facts that surprise me, these are not on the slides. In order to make up for that, at least a little bit, I interspersed the original pages with a few extra ones, offering a little bit of additional discussion.

Hard to say to what extent that succeeds. Of course, what you might possibly get from reading this text derives as much from you as it does from me. No excuses offered!



+

a blue point here?
how large is it?
does it have parts?

are there POINTS in the blue sky?

your smartphone records millions of them - they're called "pixels"
but do you **see** them?

at least in my experience i don't
the blue sky is a blue, undifferentiated area that contains no "parts" such as "points"

how large would such points be anyway?
next year's smartphone no doubt sports even more "pixels"
does that imply the blue sky gains so many more "points"?
of course not!

but if the blue sky does not contain blue points, then what is it made of?
"blue goo" perhaps?

this is the problem of the CONTINUUM in a nutshell

Euclid of Alexandria
fl. 300 BCE



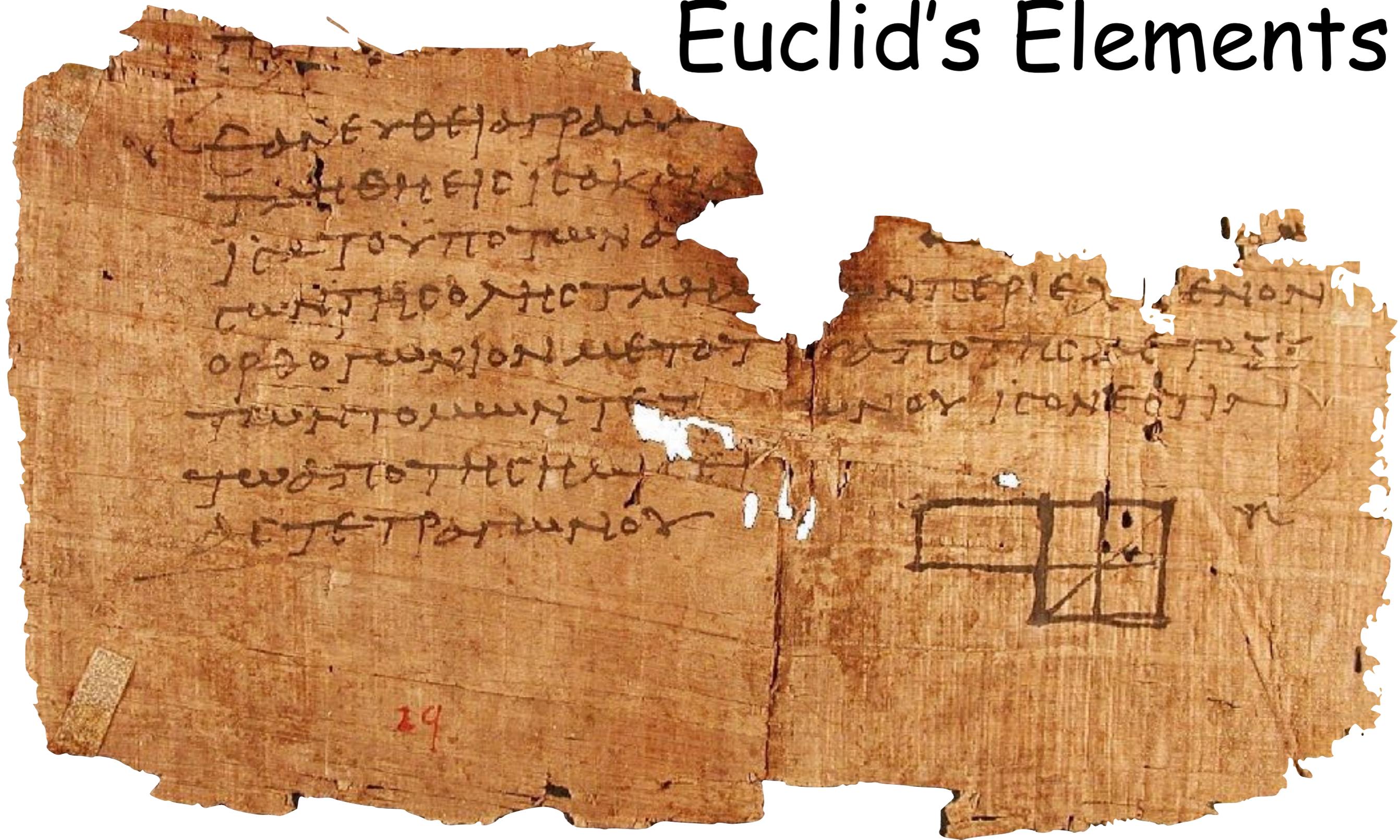
nobody has any notion of what Euclid looked like, yet there exist many "portraits"
i guess he looked like a human

there is another Euclid, a certain Euclid of Megara, a pupil of Socrates who founded the
Megarian School of Philosophy
translators in medieval times often confused him with "The Father of Geometry", author
of "The Elements", Euclid of Alexandria

we have no biographical details (the Arabian biography seems to be fictional), so it is
sometimes suggested that Euclid is a fictional character and his Elements a work due to
several authors
there is no real evidence for such a notion

Euclid wrote on various topics, here his work on optics is also of interest
unfortunately it is usually misinterpreted as a faulty treatise of the physics of radiation
or an equally faulty treatise on linear perspective
when read right the optics is an impressive formal account of the phenomenology of
vision

Euclid's Elements



we are especially interested in:

Book 1 Definition 1: "a point is that which has no parts"

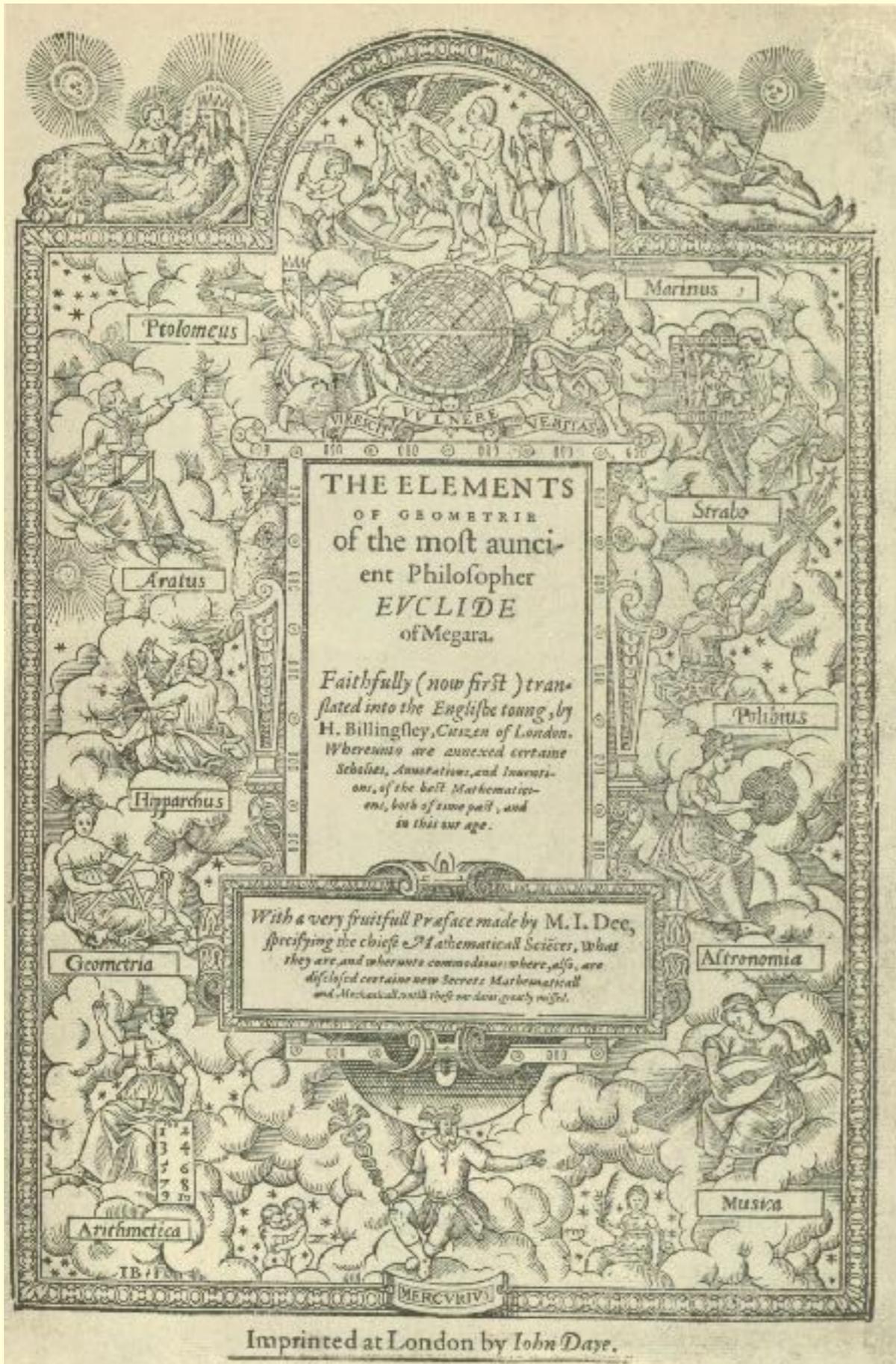
the picture shows the Papyrus Oxyrhynchus 29, a fragment of the 2nd book of the elements, a statement of the 5th proposition: "If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half"

(don't expect to understand this at first blush, it takes an effort!)

it was discovered in 1897 by Grenfell and Hunt in Oxyrhynchus and is dated late 3rd or early 4th century

the basis of modern translations is the Heiberg manuscript, discovered in 1808 at the Vatican by François Peyrard - it derives from a Byzantine workshop around 900

before that the only source was Theon's (of Alexandria) manuscript dating from the 4th century

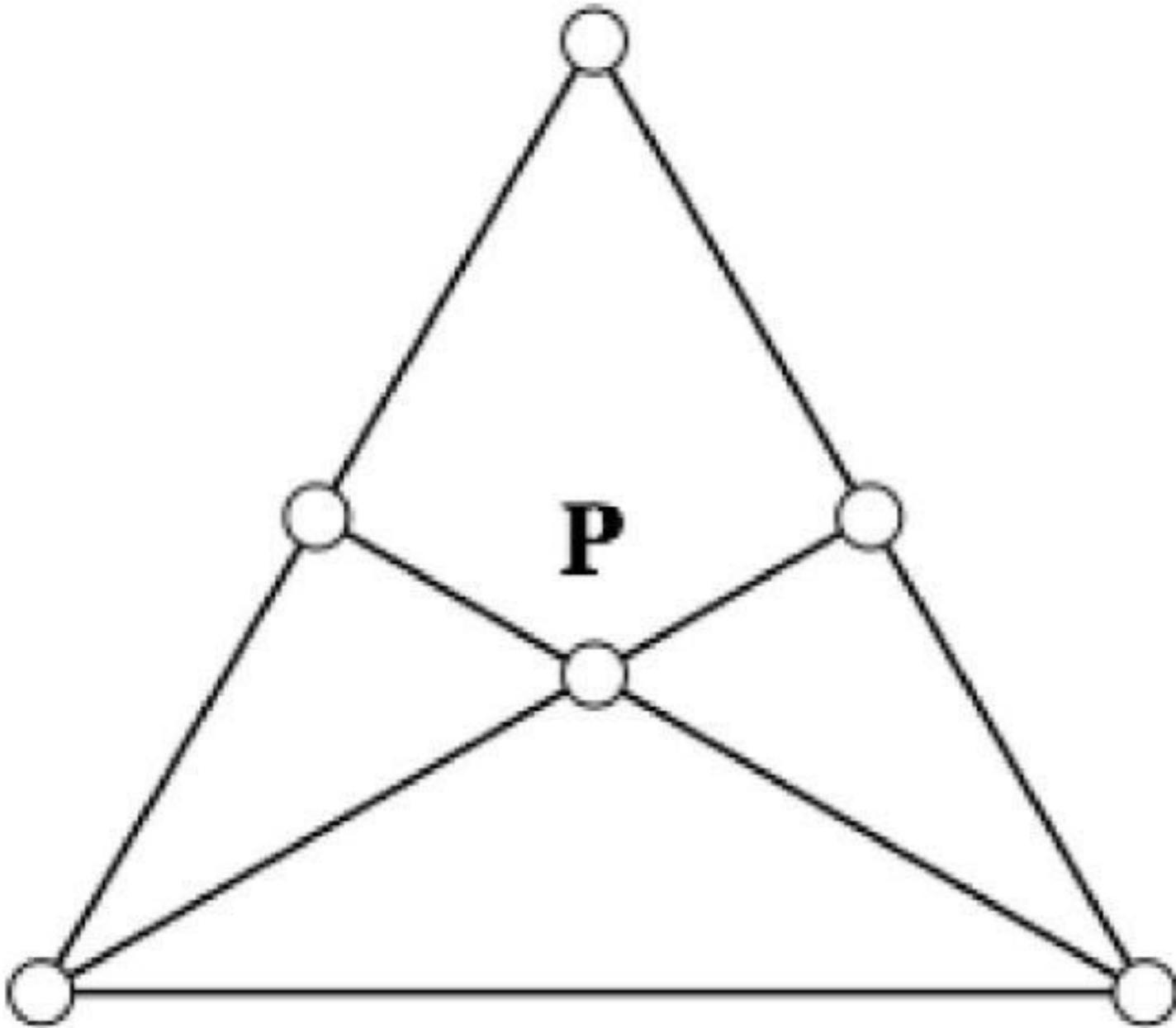


the first English version of Euclid's Elements is Sir Henry Billingsley's of 1570

Euclid has two different definitions of "point" neither of which he uses!

Euclid often merely **assumes existence**, for instance:

does "the" point *P* exist?



isn't it obvious that the lines will intersect somewhere in the interior of the triangle?

well only (perhaps) in geometries wherein the existence of planes is guaranteed, for otherwise the two lines (or "geodesics") may well turn out to be skew

as we were able to show empirically, this applies to the case of human pictorial perception

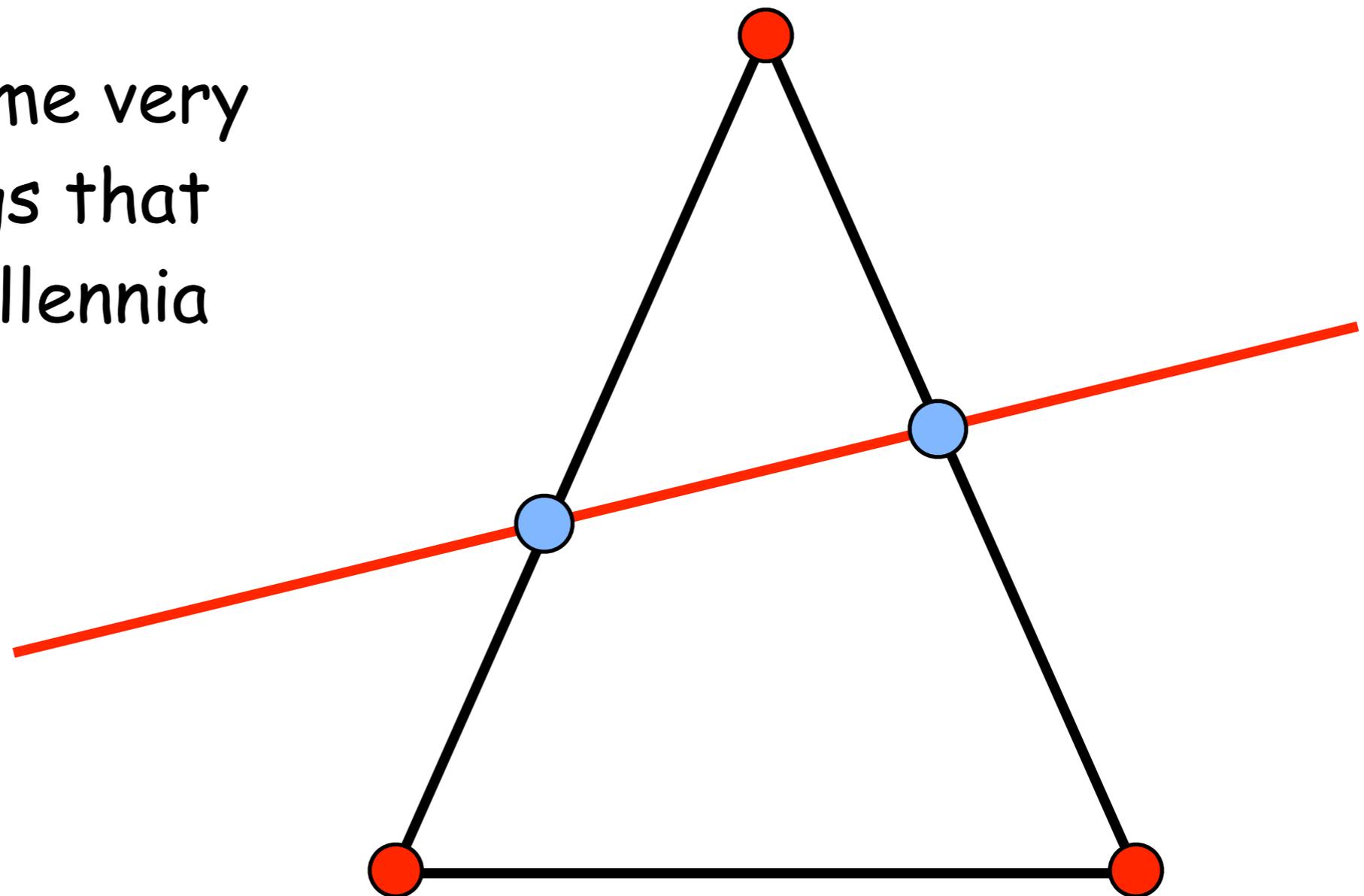
so "obvious" it is not

Euclid could have done better

Moritz Pasch (1882, *Vorlesungen über neuere Geometrie*):

"if a line, not passing through any vertex of a triangle, meets one side of the triangle then it meets another side"

Euclid missed some very elementary things that took over two millennia to rectify!



so Moritz Pasch "saved Euclid" by adding an axiom at the end of the 19th century!

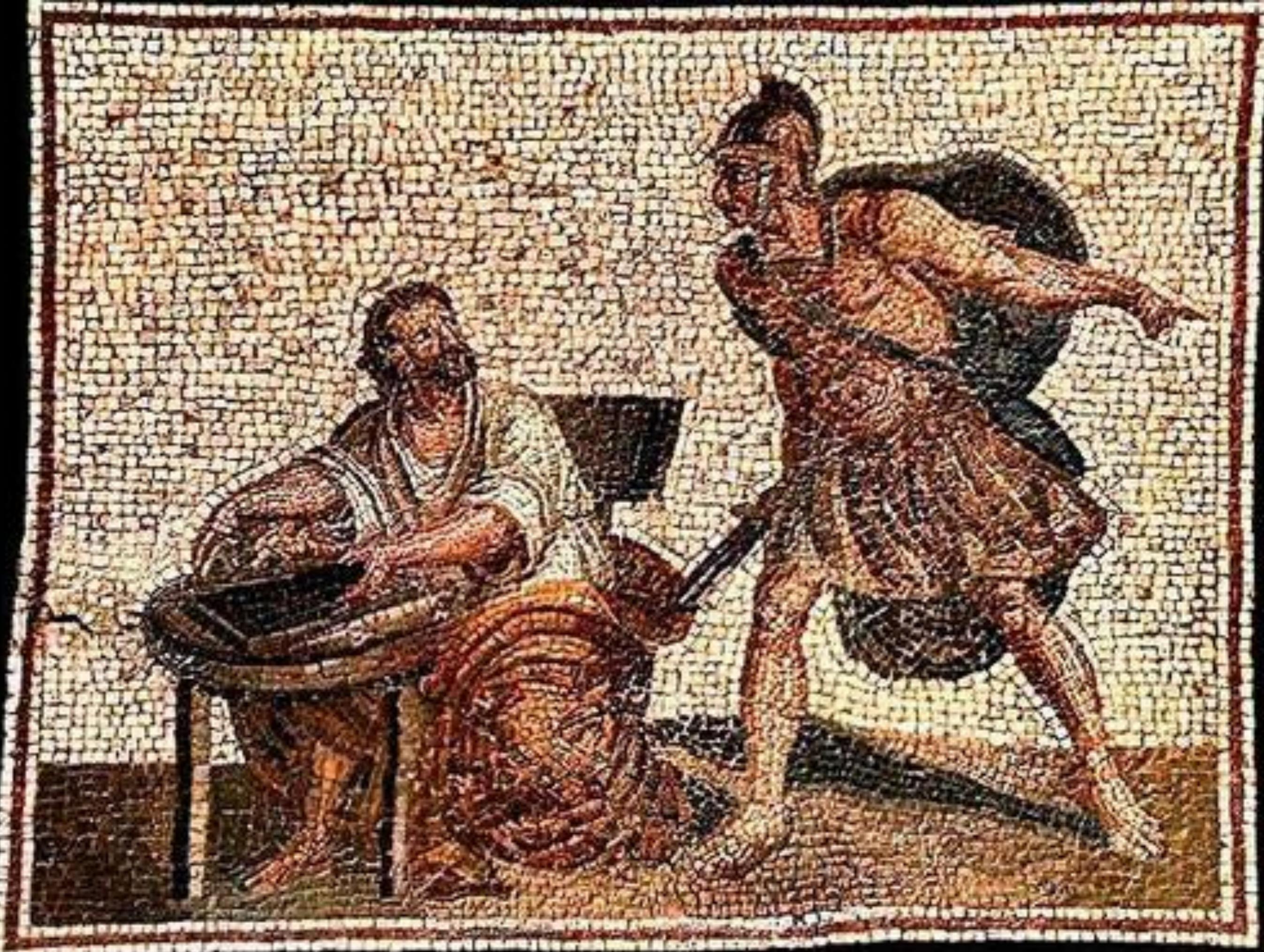
David Hilbert (1862-1943) in his "Grundlagen der Geometrie", 1902 added many more

Hilbert was more of a logician than a geometer

on the meaning of axioms he famously remarked that:

"Man muß jederzeit an Stelle von "Punkte, Geraden, Ebenen" "Tische, Stühle, Bierseidel" sagen können"

that is to say, a "point" is nothing but a fiction that satisfies the axioms

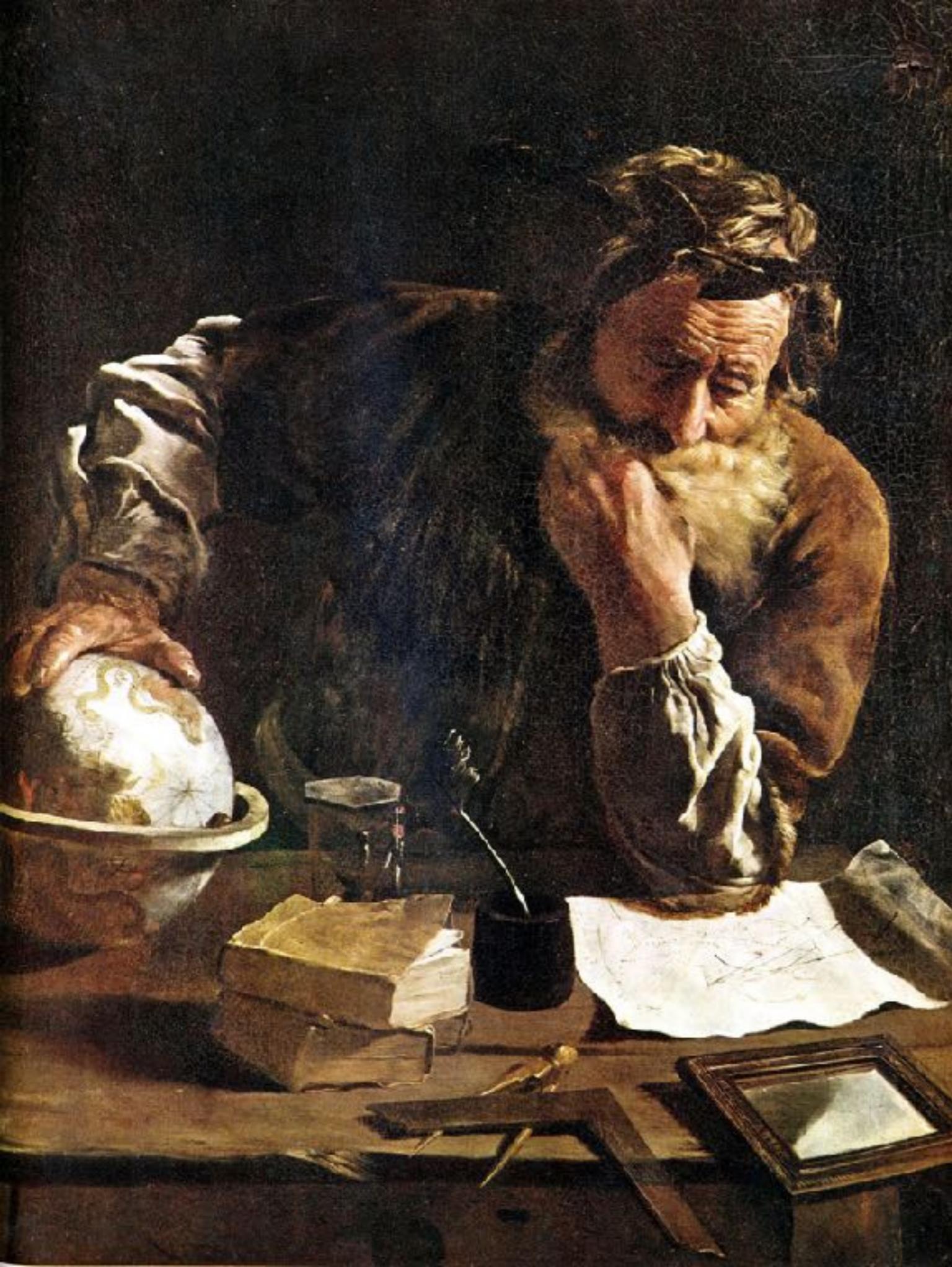


the siege of Syracuse played 214-212 BCE, the Roman army led by Marcus Claudius Marcellus

when the city fell Marcellus ordered Archimedes (then about 78 years of age) to be taken prisoner, as the mathematician had shown himself to be a first rate military engineer and might prove useful to the Roman army

Archimedes was irritated to be disturbed in his thoughts and ordered the Roman soldier out as he was involved in a geometrical problem, having drawn a figure involving circles in the sand (for lack of a blackboard)

the impatient soldier killed him on the spot - serious loss for mathematics!



Archimedes of Syracuse

c. 287 BCE

- c. 212 BCE

"any magnitude when added to itself enough times will exceed any given magnitude ..." (credited to Eudoxus of Cnidos by Archimedes)

Otto Stolz (1880's) introduced the term **Archimedean Property**

the painting is by Domenico Fetti (1620)

of course, no one knows what Archimedes might have looked like

Archimedes himself credits the "Archimedean Property" to Eudoxus of Cnidus a scholar of Plato and mathematician, astronomer and so forth, whose works are lost except for fragments in Hipparchus' commentary on Aratus' poem on astronomy

Eudoxus used geometry rather than logic or algebra to do wonderful things
consider definition 5 of the elements V, which derives from him:

"magnitudes are said to be in the same ratio, the first to the second and the third to the fourth when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order"

(don't expect to understand this at first blush, it takes an effort!)

which manages to compare certain objects without recourse to a common unit!



first step..

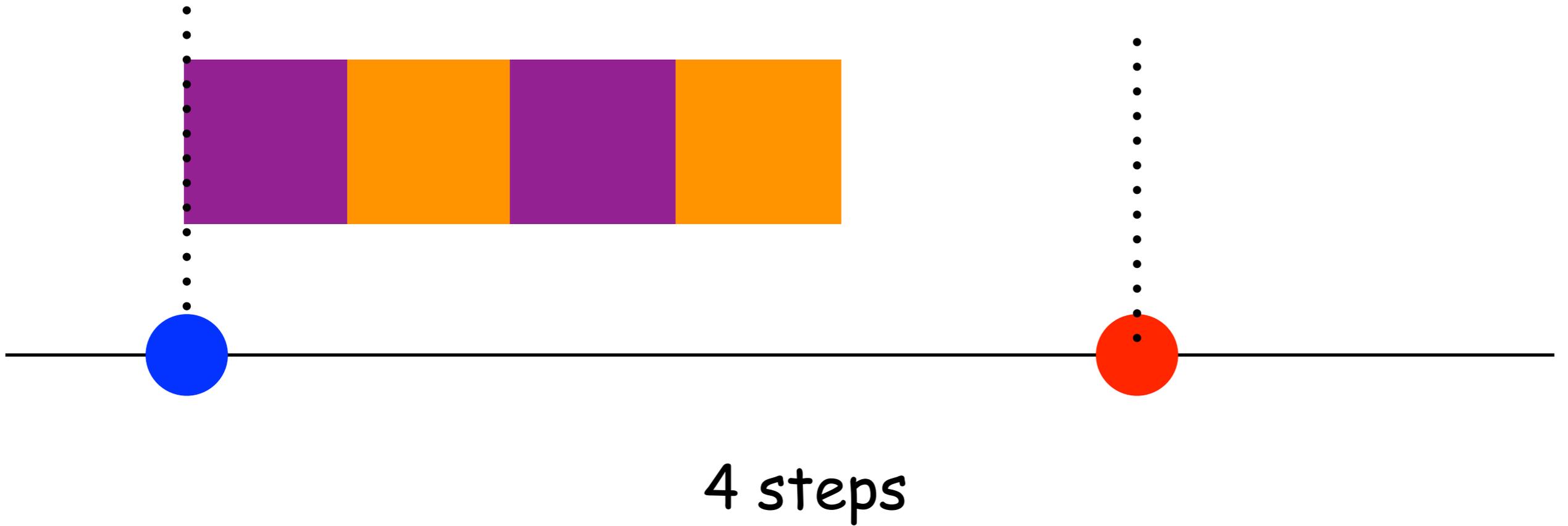
Euclid (Book V, Definition 4: "Magnitudes are said to have a ratio to one another which can, when multiplied, exceed one another" - (adopting the Archimedean Principle)



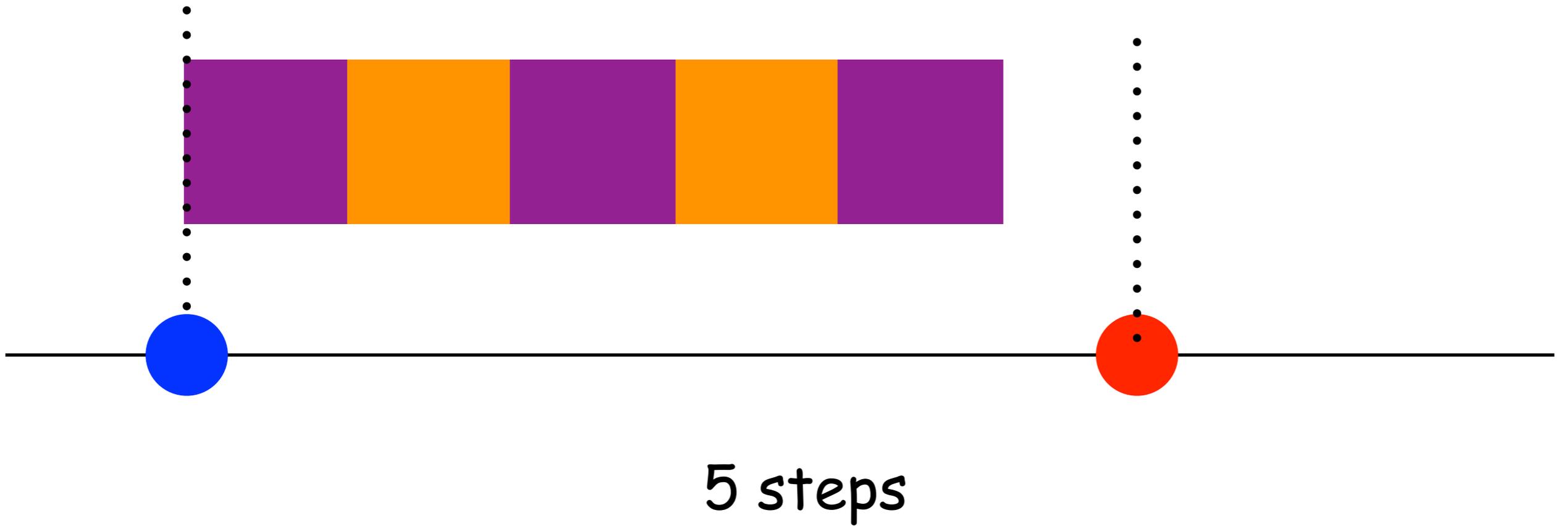
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6 steps ... **THERE!**

this was corrected by Giuseppe Veronese:

it is not necessarily the case that you'll get **ANYWHERE**,
no matter how many steps you take!



Giuseppe Veronese 1854 - 1917

FONDAMENTI DI GEOMETRIA
A PIÙ DIMENSIONI E A PIÙ SPECIE DI UNITÀ RETTILINEE
ESPOSTI IN FORMA ELEMENTARE

... it has not yet been demonstrated that there are discontinuous systems of points which satisfy all the properties of space given by experience ...

Giuseppe Veronese has many original and deep thoughts on the main topic of my talk

he was severely criticised by the mainstream mathematicians of his time (like Giuseppe Peano and the powerful Georg Cantor), being perhaps a bit too original

many of his insights apparently came from intuition first and were put in a formal framework in retrospect

the intuitions are mainly of a visual nature, it is most interesting to try to follow his thoughts from that perspective

no doubt he was a genius

his most important work is probably "Fondamenti di geometria a più dimensioni e a più specie di unità rettilinee esposti in forma elementare" of 1891

FONDAMENTI DI GEOMETRIA

A PIÙ DIMENSIONI E A PIÙ SPECIE DI UNITÀ RETTILINEE

ESPOSTI IN FORMA ELEMENTARE

LEZIONI PER LA SCUOLA DI MAGISTERO IN MATEMATICA

DI

GIUSEPPE VERONESE

PROFESSORE DELLA R. UNIVERSITÀ DI PADOVA

Siffatti tentativi di rinnovamento radicale dei principi si incontrano non di rado nella storia delle scienze. Oggi poi sono un portato naturale dello spirito critico, cui a buon diritto si vanno sempre informando le indagini scientifiche (R. BOLTZMANN: Saggio di Interpretazione della Geometria non Euclidea. - Giorn. di Battaglia, 1891).

INTRODUZIONE

PRINCIPI FONDAMENTALI DELLE FORME MATEMATICHE ASTRATTE

PARTE I.

LA RETTA, IL PIANO E LO SPAZIO A TRE DIMENSIONI NELLO SPAZIO GENERALE

PARTE II.

LO SPAZIO A QUATTRO E A N DIMENSIONI NELLO SPAZIO GENERALE

APPENDICE

BIBLIOTECA dell'ISTITUTO di FISICA
dell' UNIVERSITA' - FIRENZE

Inv. J485

DONAZIONE
DELLA RICCIA
Scienze

12
23

PADOVA

TIPOGRAFIA DEL SEMINARIO

1891

there is also a German translation of this Italian monograph:

"Grundzüge der Geometrie von mehreren Dimensionen und mehreren Arten gradliniger Einheiten in elementarer Form entwickelt"
(1894)

if you have a small number, then its square is even smaller

for instance, the square of $1/1000$ is only $1/1000\ 000$

if a number is *very* small its square must be very close to zero - what if it is zero?

let ε be a solution of $\varepsilon^2 = 0$ such that $\varepsilon \neq 0$

question: is ε positive or negative?

well, if $\varepsilon < 0$ then $\varepsilon^2 > 0$, and if $\varepsilon > 0$ then $\varepsilon^2 > 0$ too,
but both are false because $\varepsilon^2 = 0$ by assumption!

thus ε is neither positive nor negative ...

it is that close to 0!

again, let ε be a solution of $\varepsilon^2 = 0$ such that $\varepsilon \neq 0$
question: has ε the Archimedean Property?

consider $\varepsilon + \varepsilon + \dots + \varepsilon + \varepsilon$ (n times say)

apparently $n^2\varepsilon^2 = 0$ because $\varepsilon^2=0$ and n^2 just a number
thus there is **no n such that $\varepsilon+\varepsilon+\dots+\varepsilon+\varepsilon > 1!$**

ε does not have the Archimedean Property!

we have constructed a "non-Archimedean" number

in case you find this too "abstract": these numbers are the **nil-square infinitesimals** and find numerous very practical applications!

the "dual number plane" (dual numbers are the "complex" numbers $a + \varepsilon b$ with $\varepsilon^2=0$) is far more useful than the Euclidean plane in a great many scientific constructions, notably graphs of mutually incommensurate quantities (say space over time, leading to the Galilean transformation group of classical mechanics)

i have introduced "singly isotropic space", as a very natural description of pictorial space, where points in Cartesian coordinates are denoted $\{x, y, \varepsilon z\}$ - the geometry of this space has been fully explored by the early 20th c. geometers like Karl (Georg) Strubecker (1904-1991)

these are problems that relate to CONTINUITY and thus to the INFINITE and the INFINITESIMAL

the nature of "POINT" relates immediately to this

the arguments debated by the PRE-SOCRATICS are still being discussed today ... (can you believe it?)

many of the arguments are of a phenomenological (thus psychological!) nature

are lines like this?



easy to take out an item!

or like this?



cutting the string destroys the sequence!

or maybe like this?



when the chain breaks the parts still hang together

or perhaps more like this?



the smallest parts of a rope are still like ropes!

QUESTIONS

are there "smallest parts" at all, what are they like?

atoms perhaps? can they have parts?

can you remove a "point" from a line?

then what happens to it? does it fall apart?



"if a stick can be broken it never was a whole"

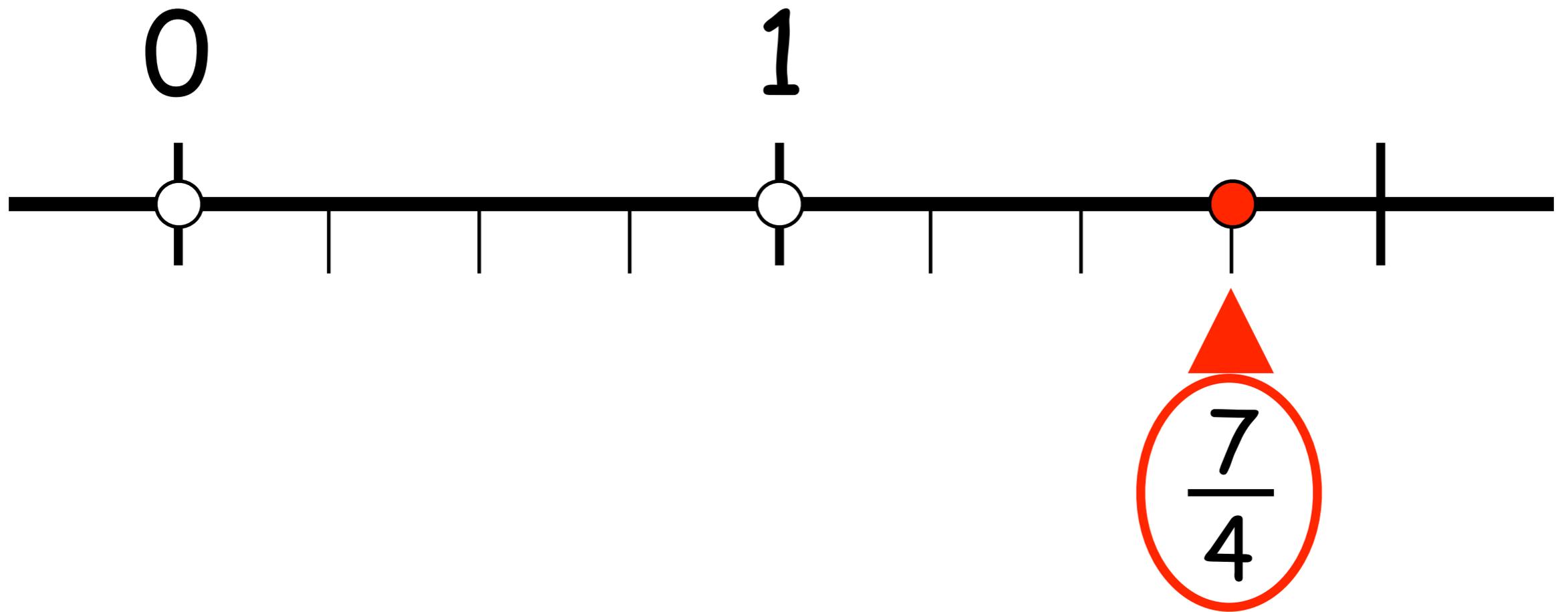
"you can't cut a line with a hatchet"

(pre-socratic philosophers)



Pythagoreans (orig. 6th c. BCE) busy at their thing

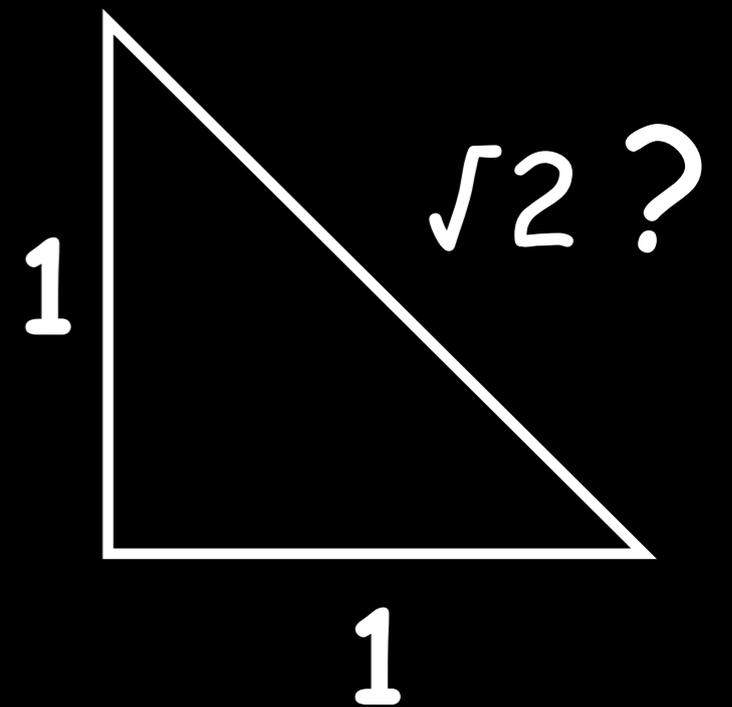
the painting "Pythagoreans celebrate the sunrise" is by Fyodor Bronnikov (1827-1902) a history and genre painter, originally from Russia, but mainly working in Italy, at Rome



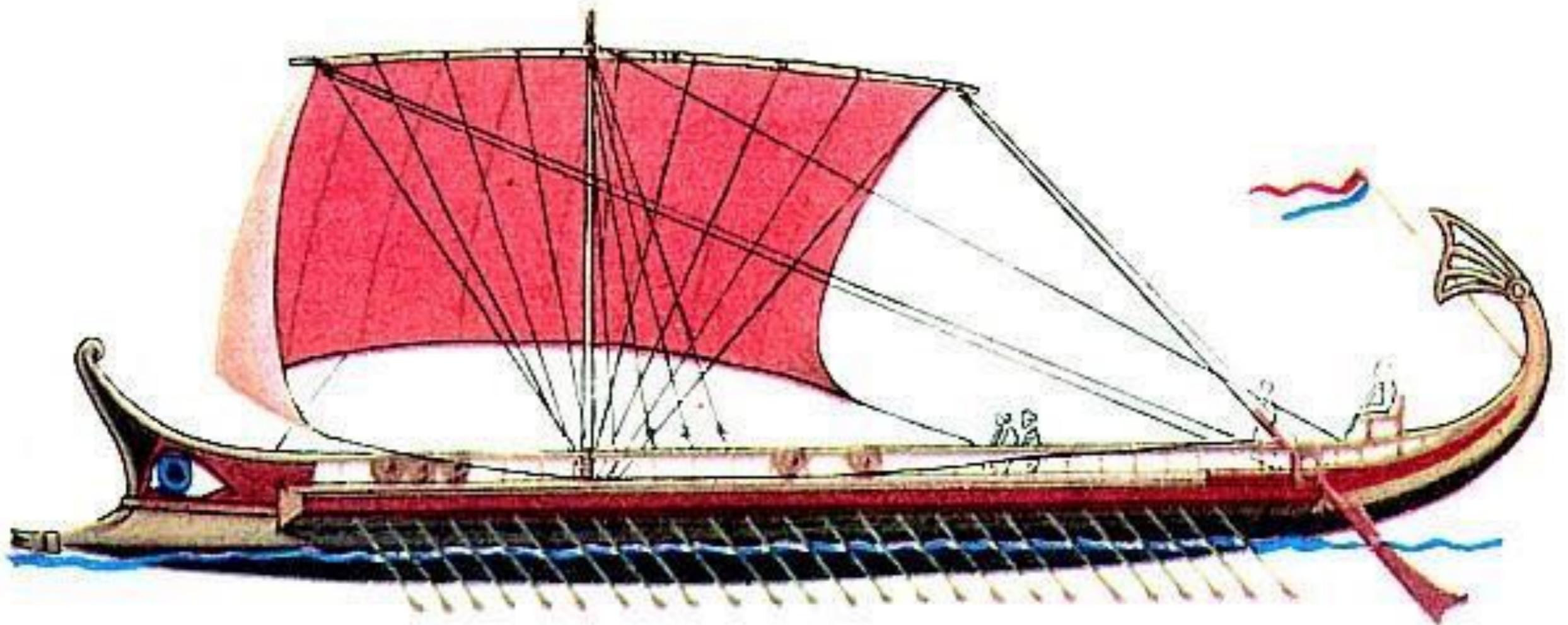
the Pythagoreans held that any point on the line could be **indicated** by a fraction of two natural numbers like $\frac{7}{4}$ (=1.75)

ca. 1700 BCE: Babylonian clay tablet defining the

△ hypotenuse as $30547 / 21600 = 1.41421296\dots$



ca. 520 BCE Hippiasus of Metapontum was brutally murdered by the Pythagoreans because he revealed an easy proof that the hypotenuse is not a Pythagorean number!



suppose $\sqrt{2} = \frac{n}{m}$ (common factors cancelled)

because $n^2 = 2m^2$ we have that n^2 is even,
thus n is also even, consequently m is odd

(for if n has a factor 2, m has not: common factors are cancelled!)

because n is even, $n = 2k$ for some k

then, because $n^2 = 2m^2$ we have $4k^2 = 2m^2$

thus $m^2 = 2k^2$ so m^2 must be even, and
consequently m is even

the hypothesis $\sqrt{2} = \frac{n}{m}$

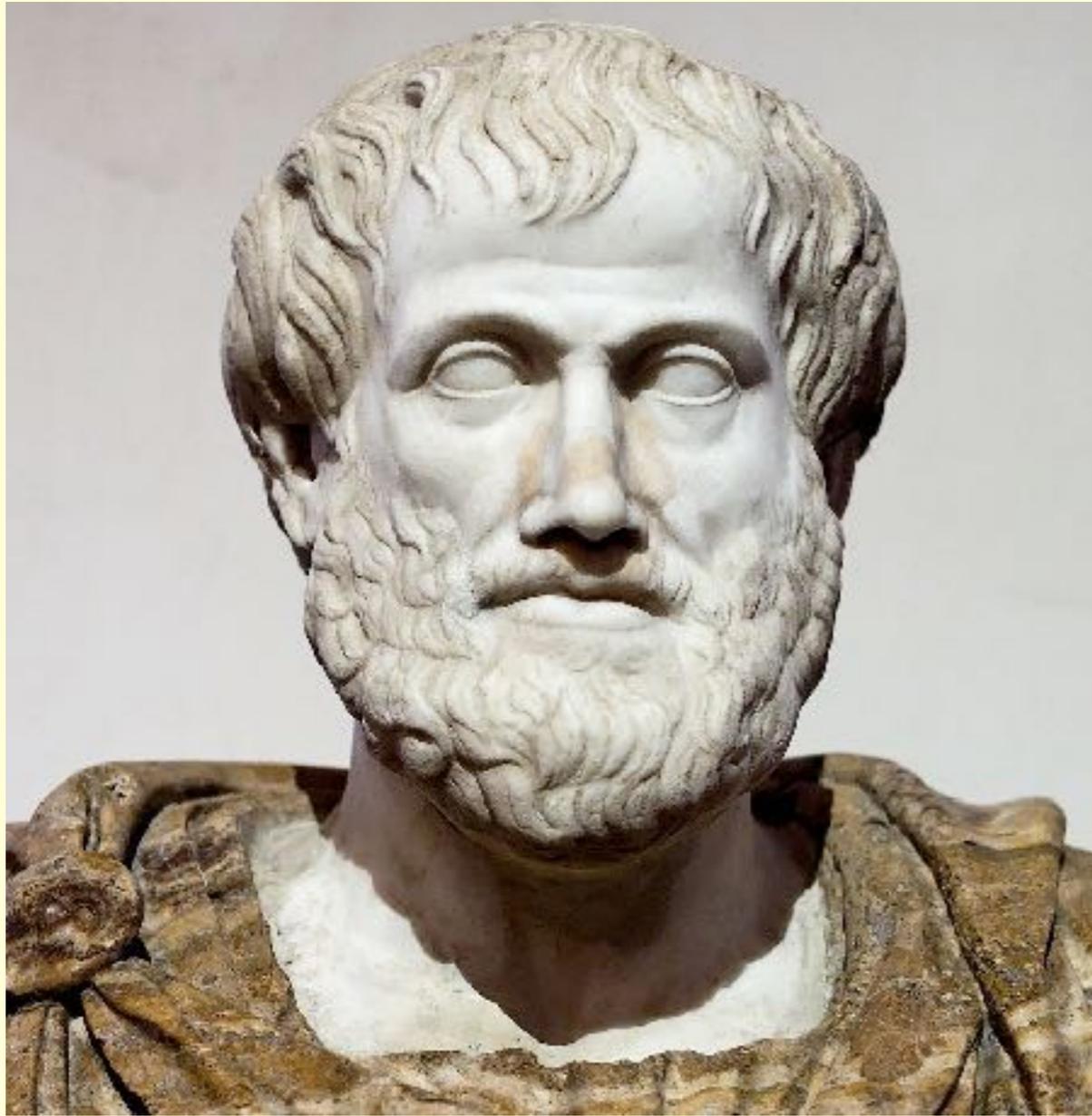
implies that m is both odd and even!

but that cannot be, thus the initial hypothesis cannot hold:

apparently $\sqrt{2}$ is not a Pythagorean number (leading to the murder of Hippasus ...)

notice the use of **TERTIUM NON DATUR!**

tertium non datur denotes the law of the excluded middle, the 3rd of the classical laws of thought: for any proposition it is the case that either that proposition is true, or that its negation is true



it was originally formulated by Aristotle in "On Interpretation"

Principia Mathematica has:

$$\star 2 \cdot 11. \quad \vdash \cdot p \vee \sim p.$$

here are a thousand decimal places of $\sqrt{2}$:

1.41421356237309504880168872420969807856967187537694807317667973799073247846210703885038
753432764157273501384623091229702492483605585073721264412149709993583141322266592750559
275579995050115278206057147010955997160597027453459686201472851741864088919860955232923
048430871432145083976260362799525140798968725339654633180882964062061525835239505474575
028775996172983557522033753185701135437460340849884716038689997069900481503054402779031
645424782306849293691862158057846311159666871301301561856898723723528850926486124949771
542183342042856860601468247207714358548741556570696776537202264854470158588016207584749
226572260020855844665214583988939443709265918003113882464681570826301005948587040031864
803421948972782906410450726368813137398552561173220402450912277002269411275736272804957
381089675040183698683684507257993647290607629969413804756548237289971803268024744206292
691248590521810044598421505911202494413417285314781058036033710773091828693147101711116
8391658172688941975871658215212822951848847

currently trillions of decimals are "known" - wasting many days of computer power - but we still are far from locating the "square root of two" on the number line

indeed, as Hippasus proved, we NEVER will!

Giambattista Vico

1668-1744

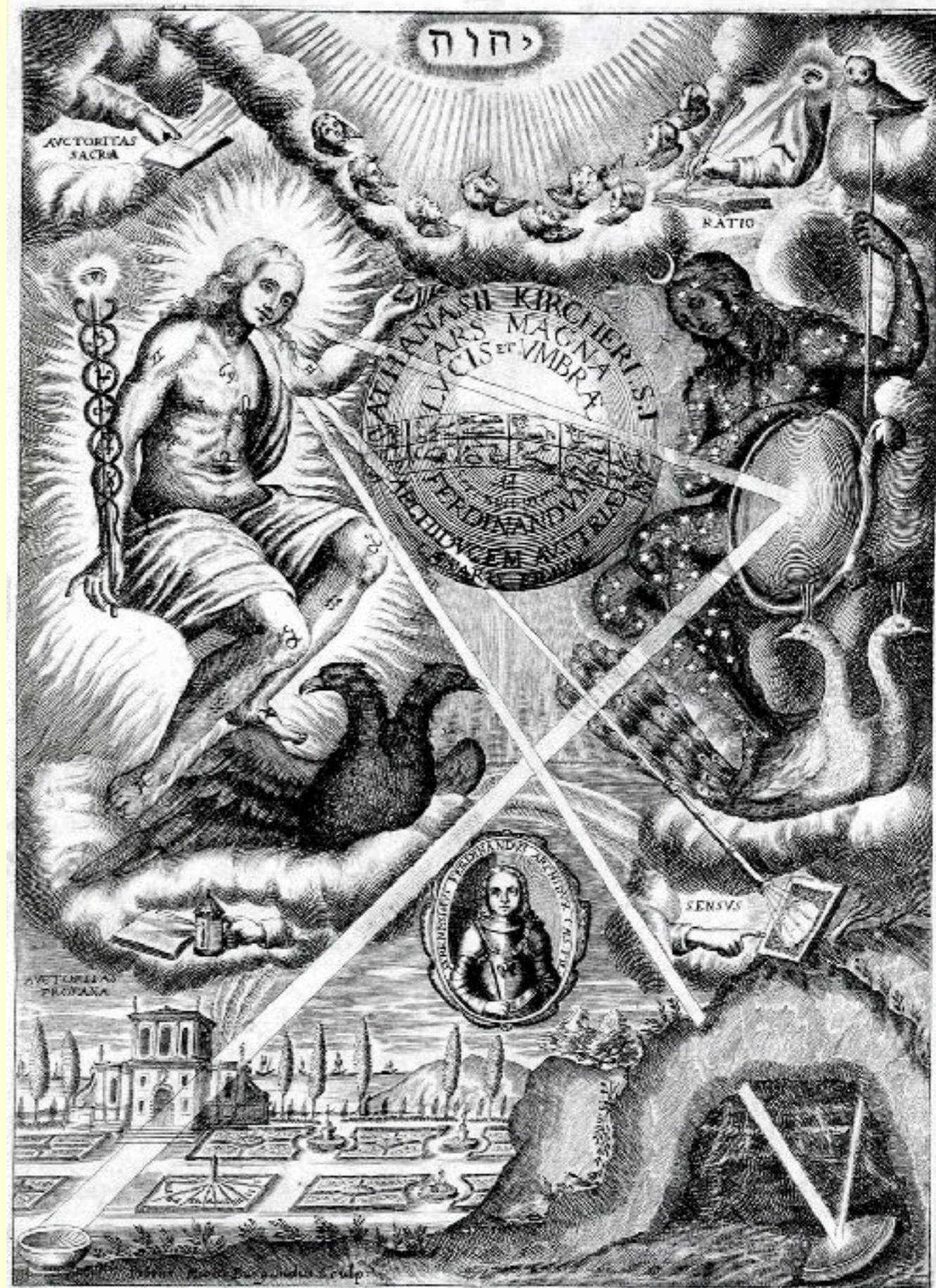


Giambattista Vico (1668-1744) was a political philosopher from the Age of Enlightenment

his work of main interest here is the "Scienza Nuova" of 1725 and his famous dictum "Verum esse ipsum factum" - the truth itself is made - he was perhaps the father of constructivist epistemology

of course he was right:

- do we understand a rabbit? of course not, never! it has neither design, nor purpose
- do we understand an alarm clock? sure, for we made it ourselves



why should this be of interest to you?

- it implies that the points on a line can neither be indicated, nor counted
- between every two Pythagorean numbers there are infinitely many "real numbers"!
- but the real numbers cannot be "known" in the sense of being constructed

most points resist being pointed out!

Giambattista Vico (1710) **VERUM FACTUM**

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Giambattista Vico (1710) **VERUM FACTUM**

Simon Stevin

1548-1620



Simon Stevin invented the decimal notation, allowing one to work with **any** number, albeit only up to a certain precision

Stevin was a great mathematician, but also an engineer, a practical man

we still use his methods on a daily basis!

as a Dutchman (i count the Flemish in, of course) i am proud of Stevin

III. VOORSTEL VAND E
MENICHVULDIGHINGHE.

Wesende ghegheven Thiendetal te Menichvuldighen, ende Thiendetal Menichvulder: haer Vytbreng te vinden.

T GHEGHEVEN. Het sy Thiendetal te Menichvuldighen $32 \textcircled{5} \textcircled{1} 7 \textcircled{2}$, ende het Thiendetal Menichvulder $89 \textcircled{4} \textcircled{1} 6 \textcircled{2}$. T BEGHEERDE. Wy moeten haer Vytbreng vinden.

WERCKING. Men sal de gegevē getalē in oirden stellen als hier nevē, Menichvuldigende naer de gemeene maniere van Menichvuldighen met heele ghetalen aldus:

	$\textcircled{5} \textcircled{1} \textcircled{2}$
	3 2 5 7
	8 9 4 6
	1 9 5 4 2
	1 3 0 2 8
	2 9 3 1 3
	2 6 0 5 6
	2 9 1 3 7 1 2 2
	$\textcircled{4} \textcircled{1} \textcircled{2} \textcircled{3} \textcircled{4}$

Gheeft Vytbreng (door het 3^e. Prob. onser Fran. Arith.) 29137122: Nu om te weten wat dit sijn, men sal vergaderen beyde de laetste gegeven teekenen, welcker een is $\textcircled{2}$, ende het ander oock $\textcircled{2}$, maecken tsamen $\textcircled{4}$, waer uyt men besluyten sal, dat de laetste cijffer des Vytbrengs is $\textcircled{4}$, welke bekent wesende soo sijn oock (om haer volghende oirden) openbaer alle dander, Inder voughen dat $2913 \textcircled{7} \textcircled{1} 1 \textcircled{2} 2 \textcircled{3} 2 \textcircled{4}$, sijn het begheerde Vytbreng. BEWYS, Het ghegheven Thiendetal te menichvuldighen $32 \textcircled{5} \textcircled{1} 7 \textcircled{2}$, doet (als

blijct

blijct door de derde Bepaling) $32 \frac{5}{10} \frac{7}{100}$, maecken tsamen $32 \frac{57}{100}$; Ende door de selve reden blijct den Menichvulder $89 \textcircled{4} \textcircled{1} 6 \textcircled{2}$, weerdich te sijne $89 \frac{46}{100}$, met de selve vermenichvuldicht de voornoemde $32 \frac{57}{100}$, gheeft Vytbreng (door het 12^e. probleme onser Franscher Arith.) $2913 \frac{7122}{10000}$; Maer soo veel is oock weerdich den voornoemden Vytbreng $2913 \textcircled{7} \textcircled{1} 1 \textcircled{2} 2 \textcircled{3} 2 \textcircled{4}$, het is dan den waren Vytbreng; Twelck wy bewijsen moesten. Maer om nu te bethoonen de reden waerom $\textcircled{2}$ vermenichvuldicht door $\textcircled{2}$, gheeft Vytbreng (welck de somme der ghetalen is) $\textcircled{4}$. Waerom $\textcircled{4}$ met $\textcircled{5}$, geeft Vytbreng $\textcircled{9}$, ende waerom $\textcircled{9}$ met $\textcircled{3}$ gheeft $\textcircled{3}$, etc. soo laet ons nemen $\frac{2}{10}$ ende $\frac{3}{100}$ (welcke door de derde Bepalinghe sijn $2 \textcircled{1} 3 \textcircled{2}$) hare Vytbreng is $\frac{6}{1000}$, welke door de voornoemde derde Bepalinge sijn $6 \textcircled{3}$. Vermenichvuldighende dan $\textcircled{1}$ met $\textcircled{2}$, den Vytbreng sijn $\textcircled{3}$. BESLUYT. Wesende dan gegeven Thiendetal te Menichvuldighen, ende Thiendetal Menichvulder, wy hebben haren Vytbreng gevonden; als voorghenomen was gedaen te worden.

MERCKT.

Soo het laetste teecken des Thiendetals te Menichvuldigē ende Menichvulders ongelijck waren, als by exempel deen $3 \textcircled{4}$ $7 \textcircled{5} 8 \textcircled{6}$, dander $5 \textcircled{1} 4 \textcircled{2}$; Men sal doen als vooren, ende de ghesteltheit der letteren vande Werckinghe sal soodanich sijn:

	$\textcircled{4} \textcircled{5} \textcircled{6}$
	3 7 8
	5 4 $\textcircled{2}$
	1 5 1 2
	1 8 9 0
	2 0 4 1 2
	$\textcircled{4} \textcircled{5} \textcircled{6} \textcircled{7} \textcircled{8}$

B IIII.

in case you don't read Dutch:

here Stevin explains how to use decimal notation so as to manipulate any numbers for practical purposes

this may perhaps be more engineering than mathematics

from my perspective that makes sense, because as a Dutchman i know that we stand in deep shit (or clay) **all the time**

Luitzen Brouwer

1861-1966

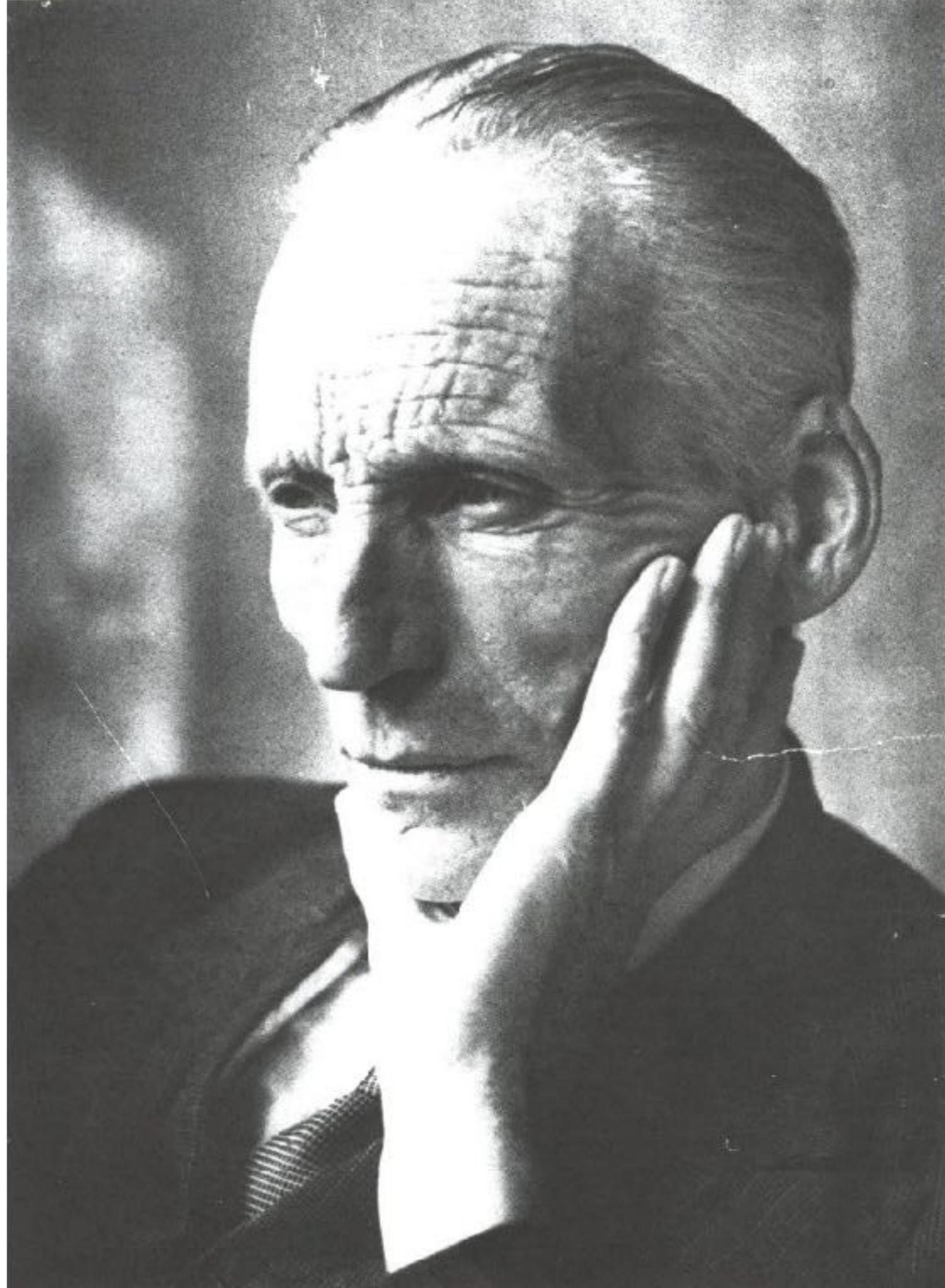
mathematics is a mental
CREATION

VERUM FACTUM

truth is mental construction

TERTIUM NON DATUR

is not acceptable



another Dutchman i am proud of!

Luitzen Brouwer singlehandedly demolished mathematics and reconstructed it in a novel way, against all odds and outcries of his mainstream colleagues

(Dutchmen had hard heads, although we are softening up)

Brouwer (1945) introduced the **CREATIVE SUBJECT**, defining real numbers as (never completed!) "choice sequences", such as

3.1415926535897932385 ... up to you(!) ...

such numbers (points on a line!) are forever in a state of **BECOMING**

did he turn mathematics into psychology?

Brouwer (1945) introduced the CREATIVE SUBJECT, defining real numbers as (never completed!) "choice sequences", such as

3.1415926535897932385 ... up to you(!) ...

such numbers (points on a line!) are forever in a state of BECOMING

did he turn mathematics into psychology?

(ASIDE ...) down to earth people like

Archimedes

Stevin

Brouwer

Vico

...

understood that science can't be left to philosophers & so forth: in science we're standing on uncertain ground, all the time!

consider: Johann Heinrich Lambert, in 1761
famously proved that

$$\pi$$

is irrational, but it is not known whether

$$\pi^{\sqrt{2}}$$

is irrational...

it might be a Pythagorean number!

exciting? to Brouwer mere sophistry!



Franz Clemens Brentano

1838 Boppard
- 1917 Zürich

... **intentional inexistence** is characteristic exclusively of mental phenomena ...

... no physical phenomenon exhibits anything like it ...

Franz Brentano is really the father of psychology

his "intentional inexistence" defines the mental realm

he took his leads from Aristotle and medieval philosophers

nowadays many psychologists understand the mental as an epiphenomenon of electrochemical processes in brain tissue, they thereby degrade psychology to physiology

Brentano also had intuitions concerning points and the continuum
these were close to Veronese's but far removed from the mainstream
mathematics of his time (Cantor, Dedekind, ...)

at the turn of the 19th c. Brentano (father of psychology!) fought the mathematics establishment from purely phenomenological perspectives

his notion of "point" was close to Brouwer's, but was given an intuitive - that is geometrical - meaning (like Veronese)

his insights were based on the study of "parts" of such entities as "points"

what is the color of the point
on the edge?



here are some suggestions:

- the point is either **red** or **blue** but only the eye of eternity knows which
- there is no fact of the matter
- the point is both **red** and **blue**
- there are no such points
- ... up to you (be creative!)

think about it!

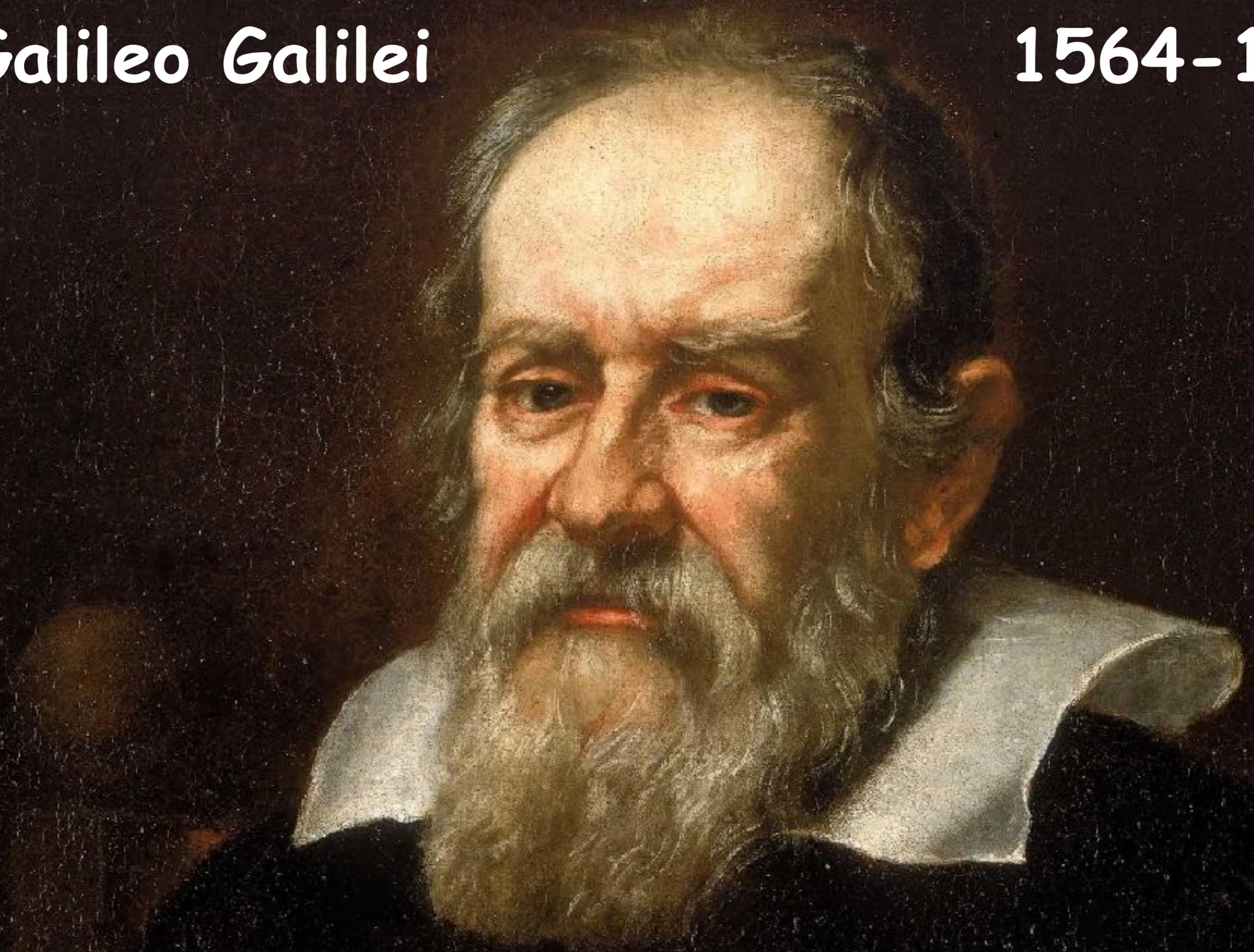
do you have a preference?

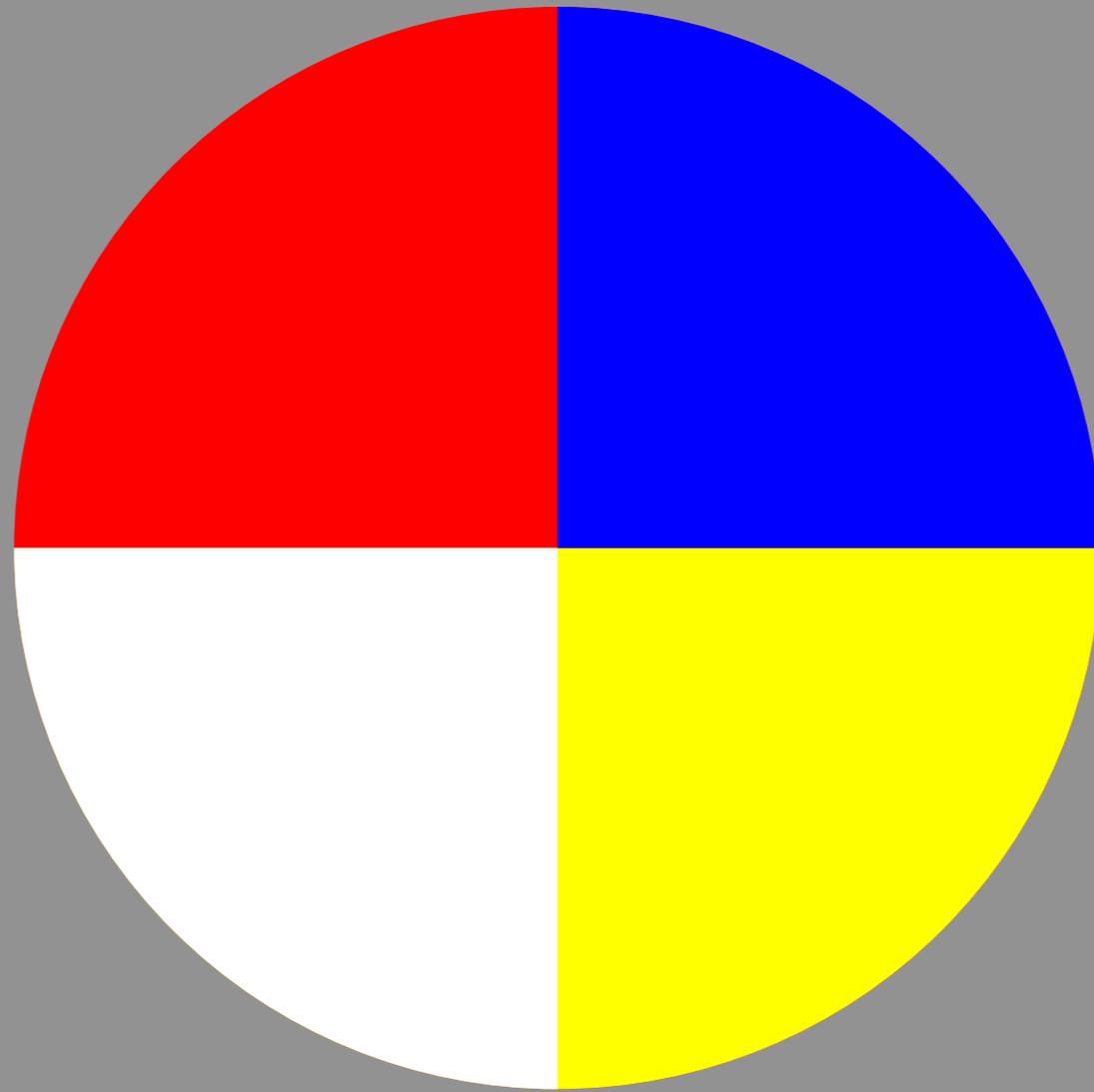
do you have other ideas - can you make these explicit?

try!

Galileo Galilei

1564-1642

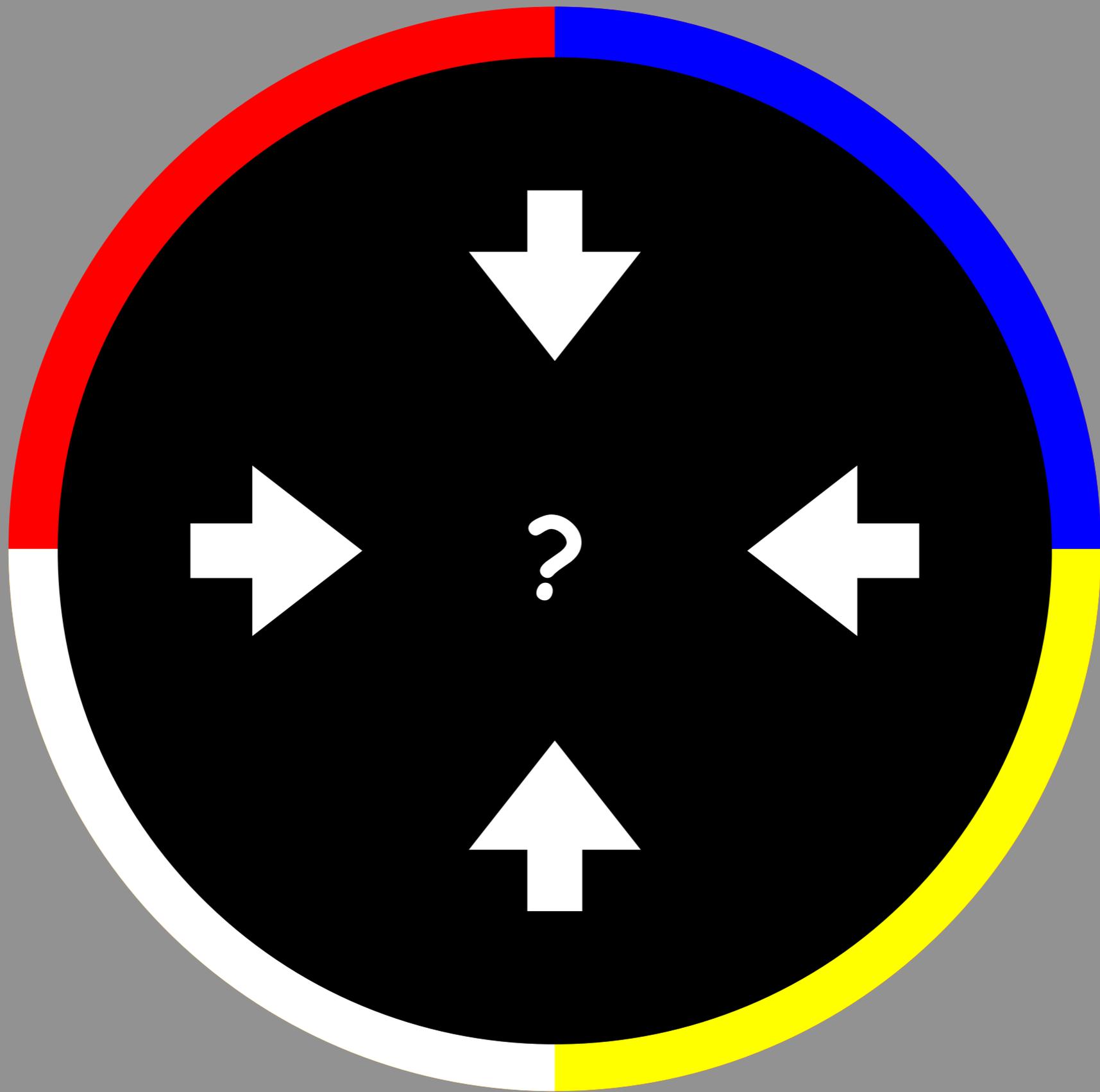




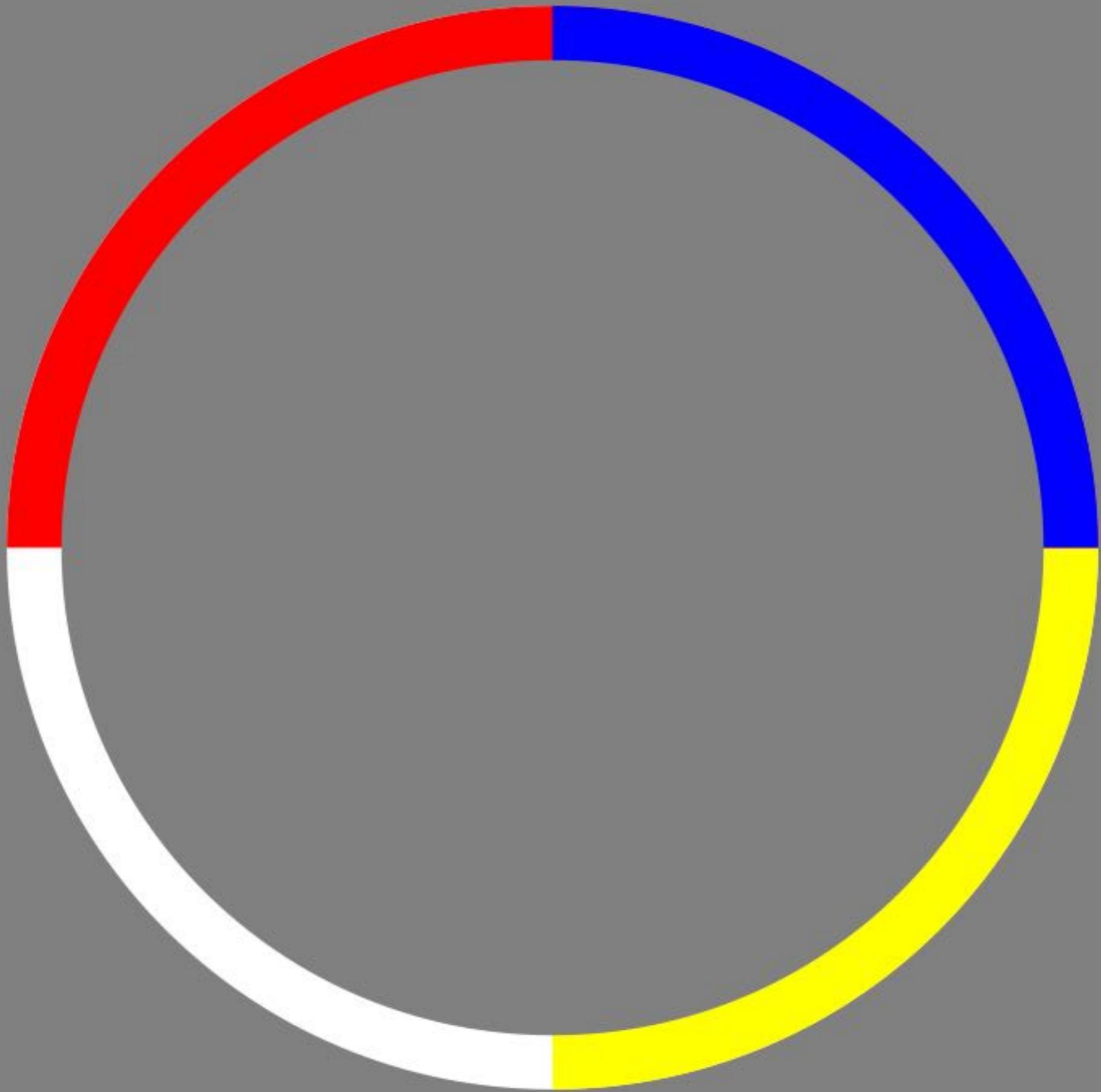
what is the color of the centre?

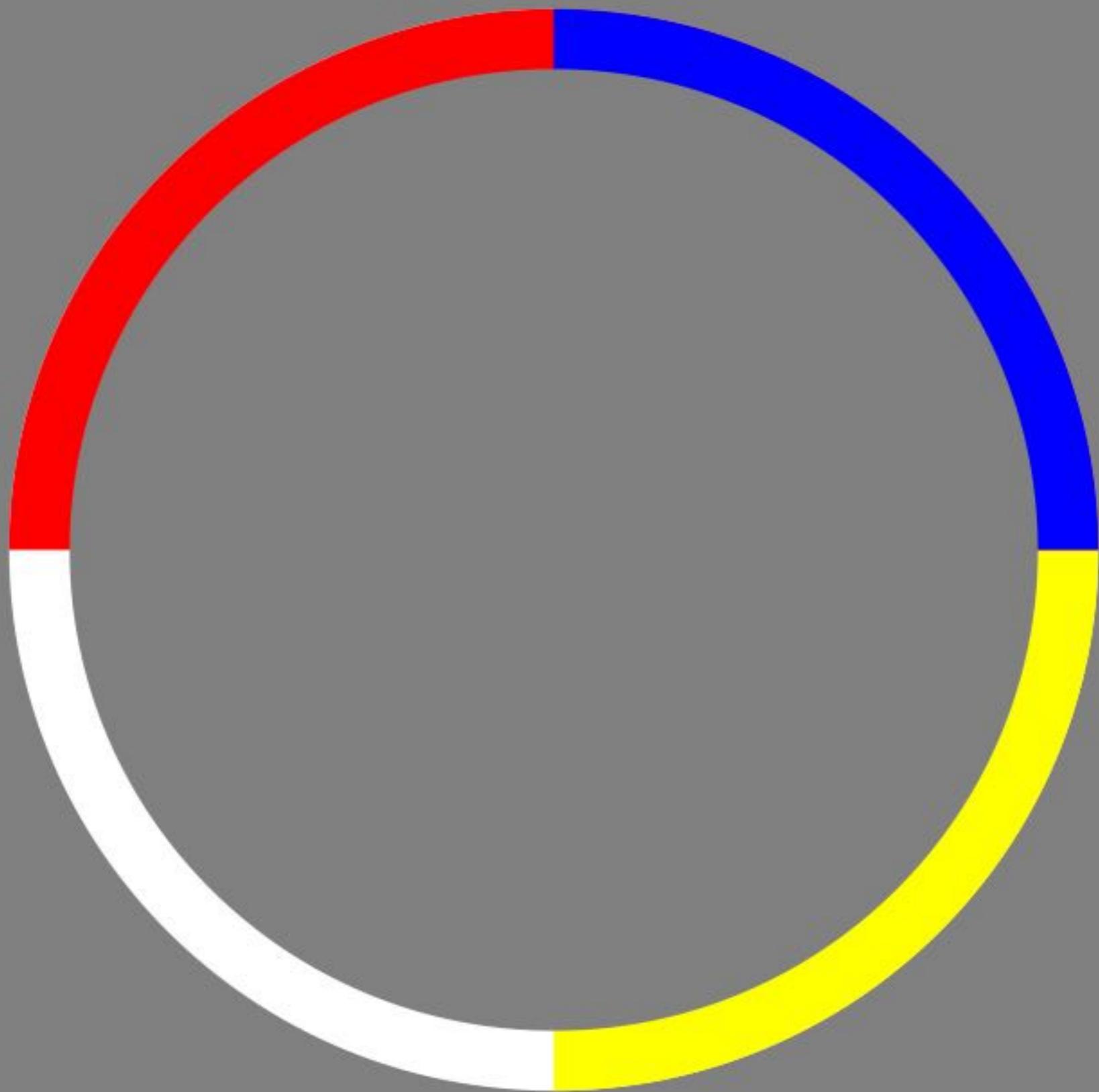
Galileo has: red, blue, white and yellow!

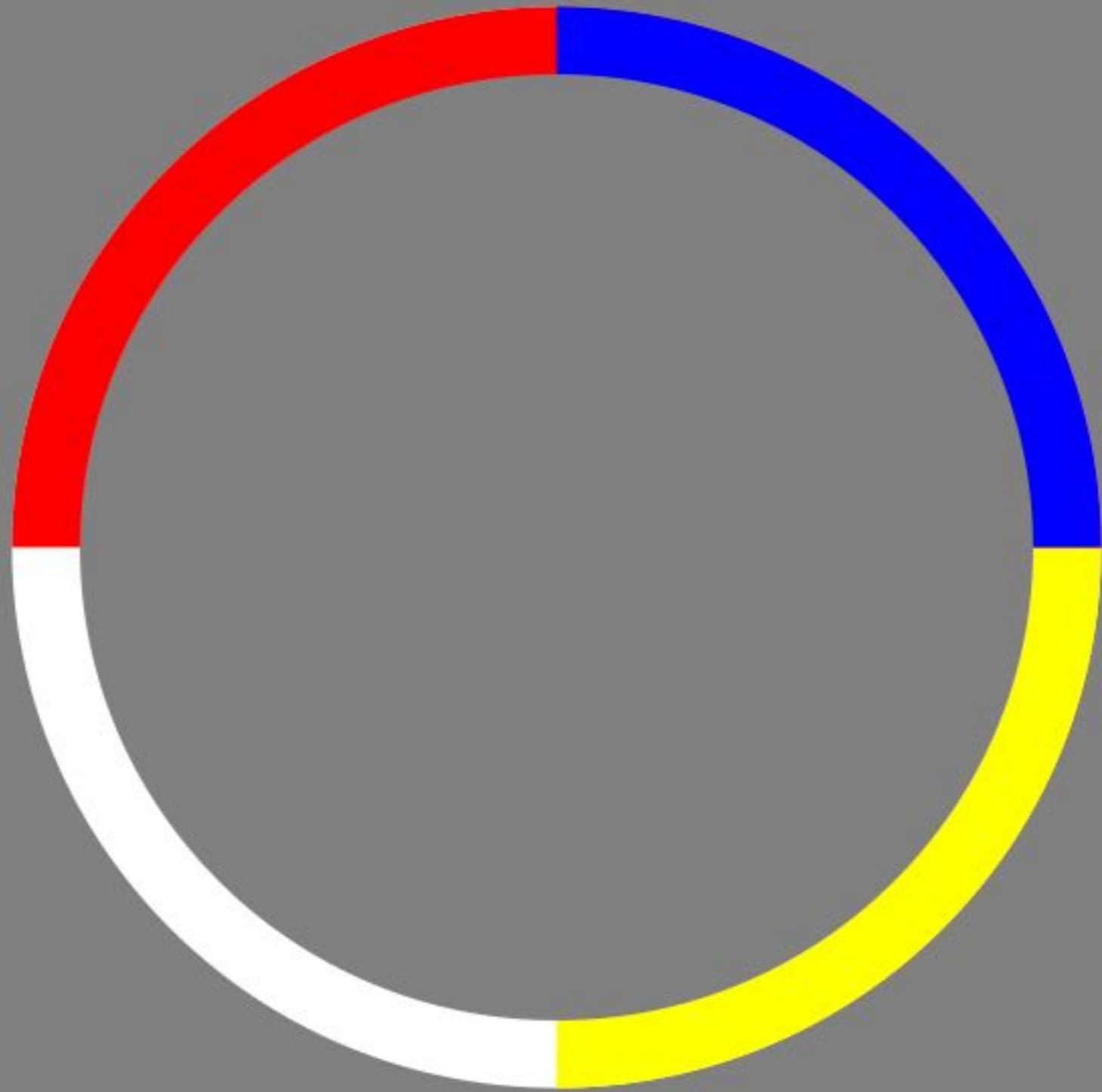
Brentano considers this "obvious" from a phenomenological perspective

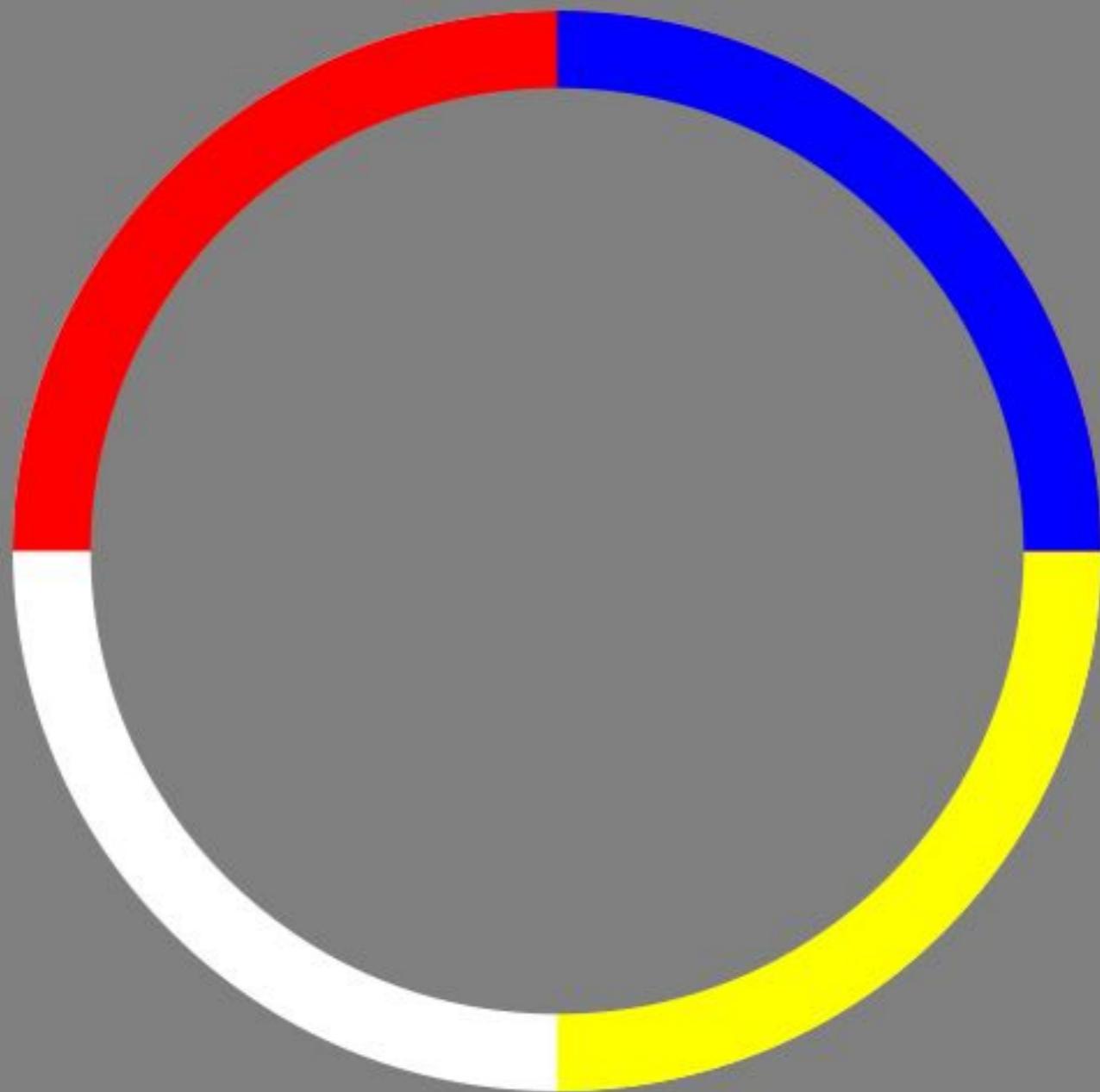


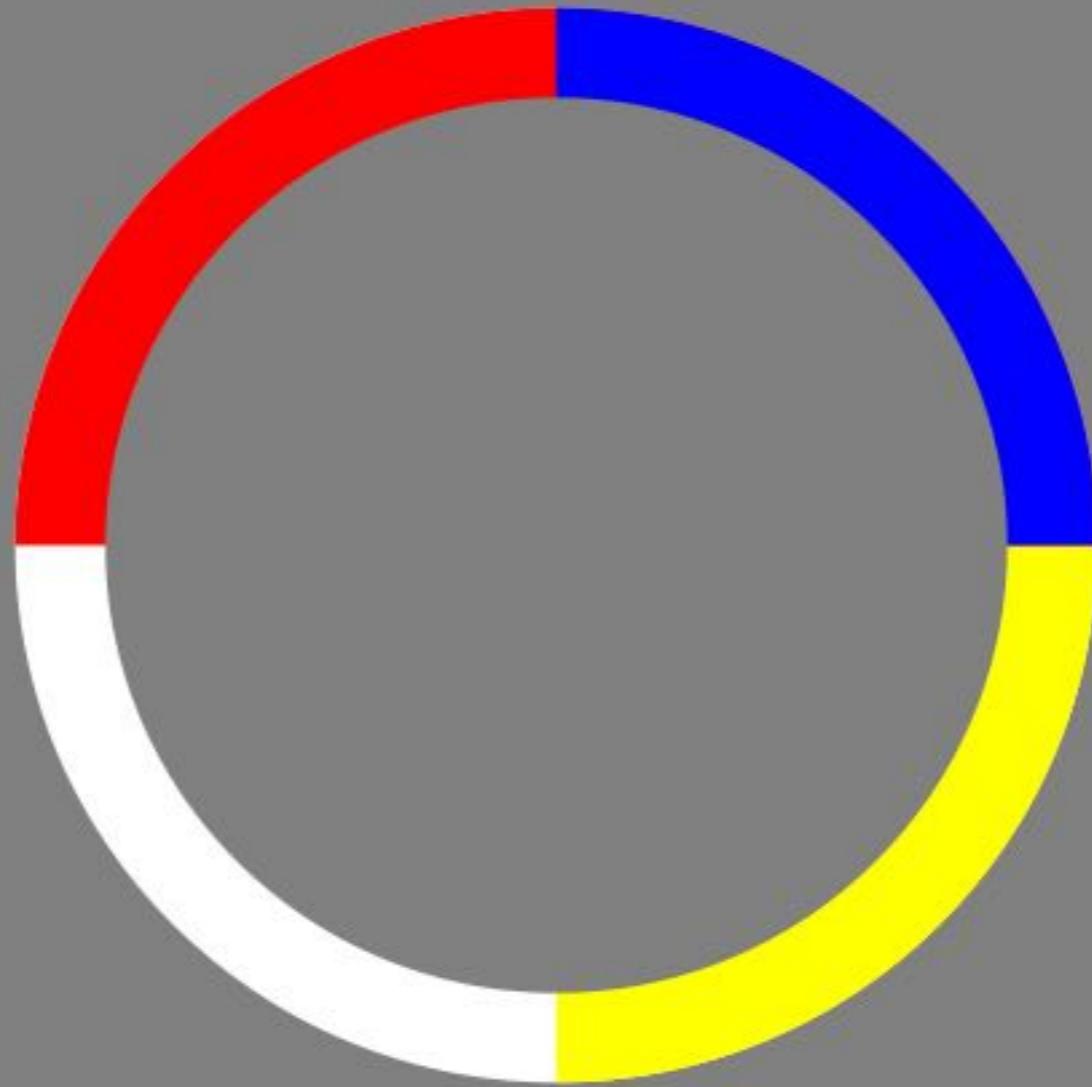
the idea is that the centre is like the circumference

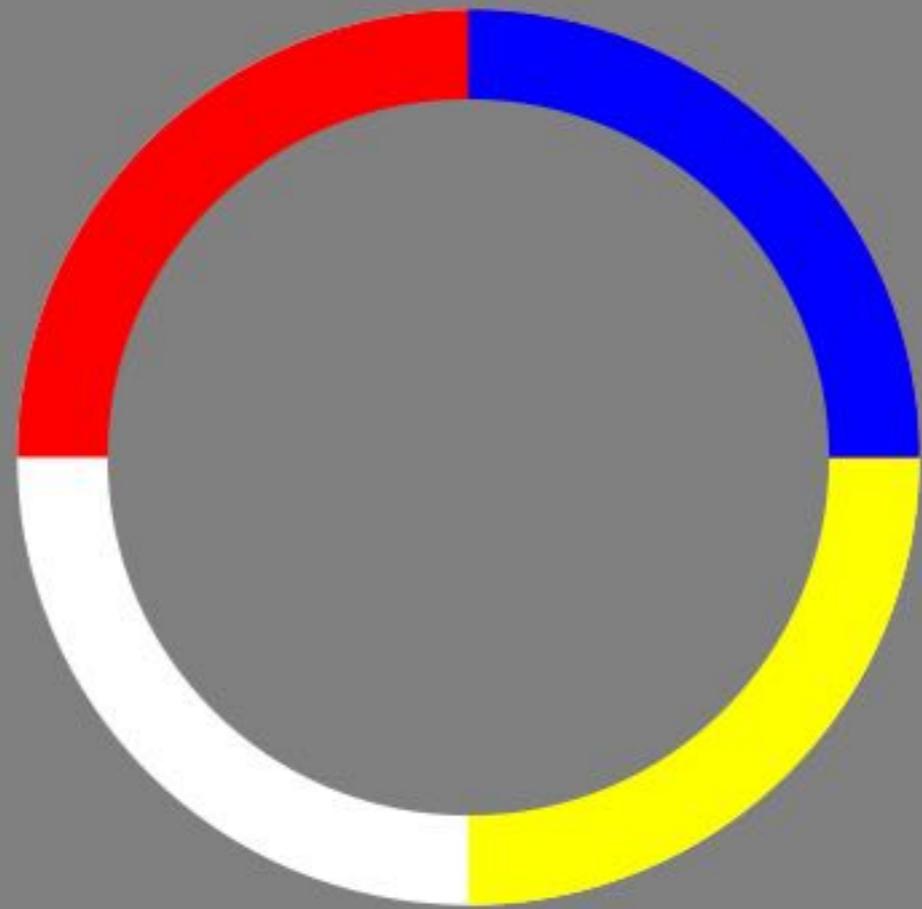


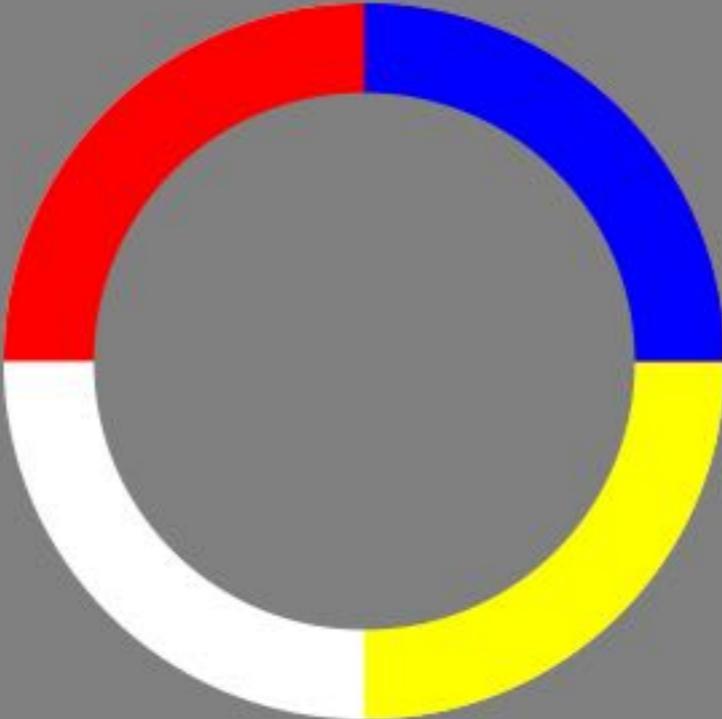


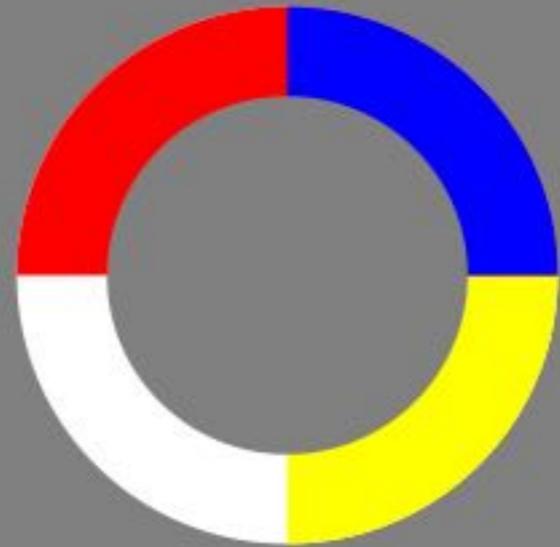








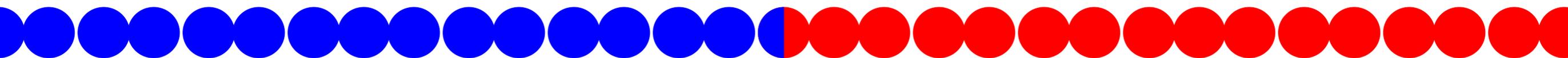








et voilà!



something like this then?

NO! for any part of the line is line-like!

the edge point belongs BOTH to the red
AND the blue side - it cannot be removed
from the line

it is a "two-sided" point, Brentano's notion
of "plerosis" (something like "fullness")



the point has a two-sided plerosis - it is part of the **red** side just as much as it is part of the **blue** side

yet it is a **SINGLE** point, or perhaps an atomic coincidence of **red** and **blue** points

points are *ATOMIC* entities (Euclid: they have no "parts")

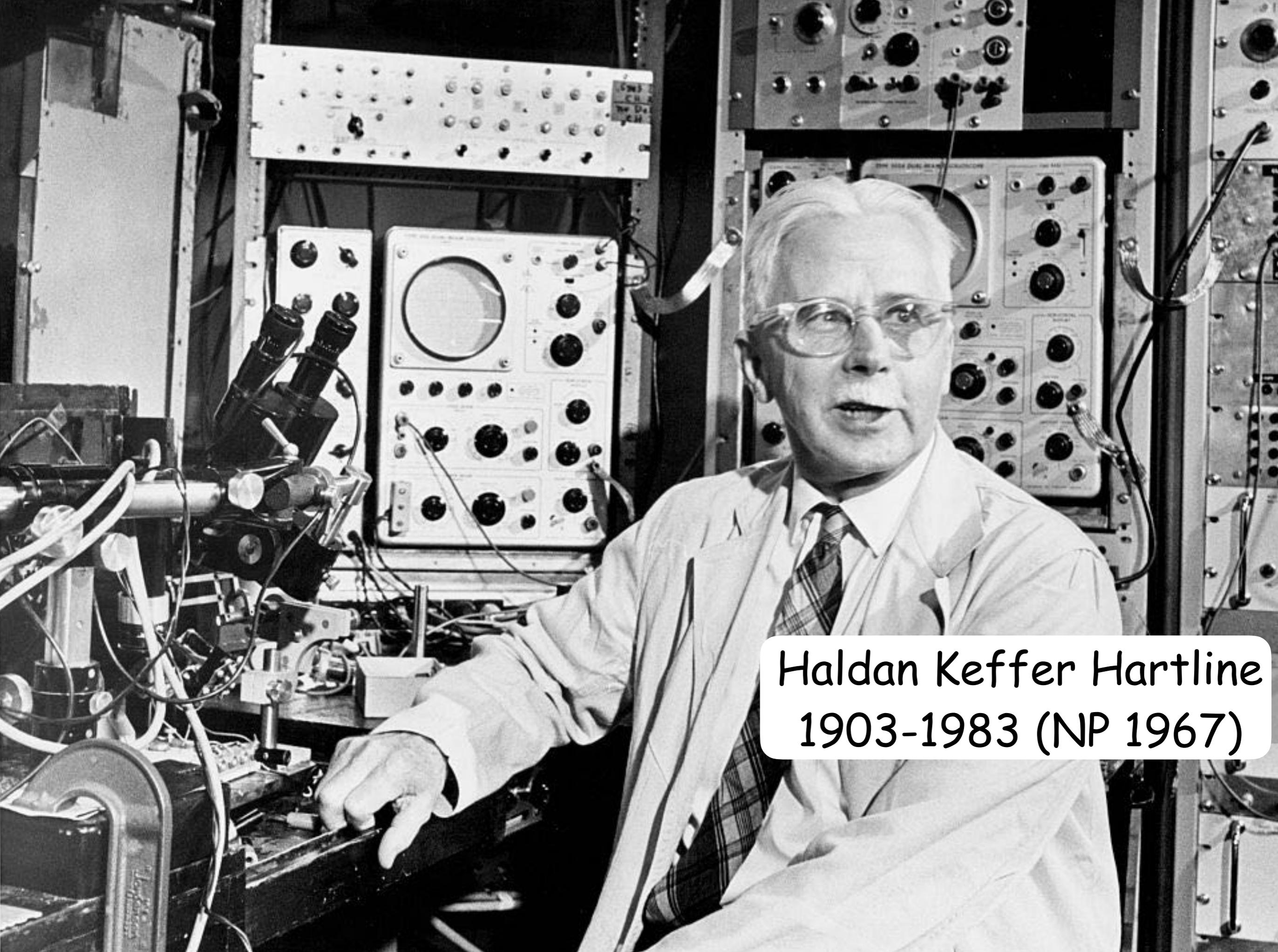
but in the plerosis notion they have "sides"

the edge point belongs just as much to the red as to the blue side,
indeed both sides need to exist for the edge point to exist

there is no such a thing as an isolated edge point

an edge point cannot be "removed"

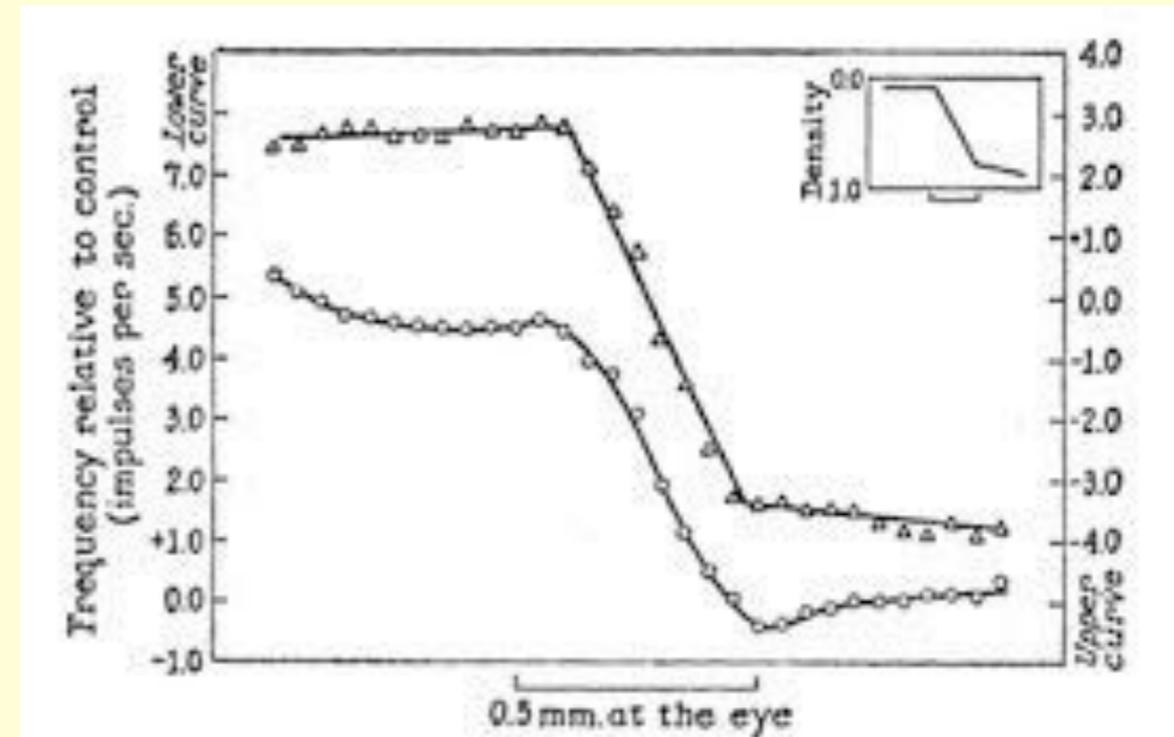
i will refer to such "sided points" as *PERCEPTIVE FIELDS* (PFs)



Haldan Keffer Hartline
1903-1983 (NP 1967)

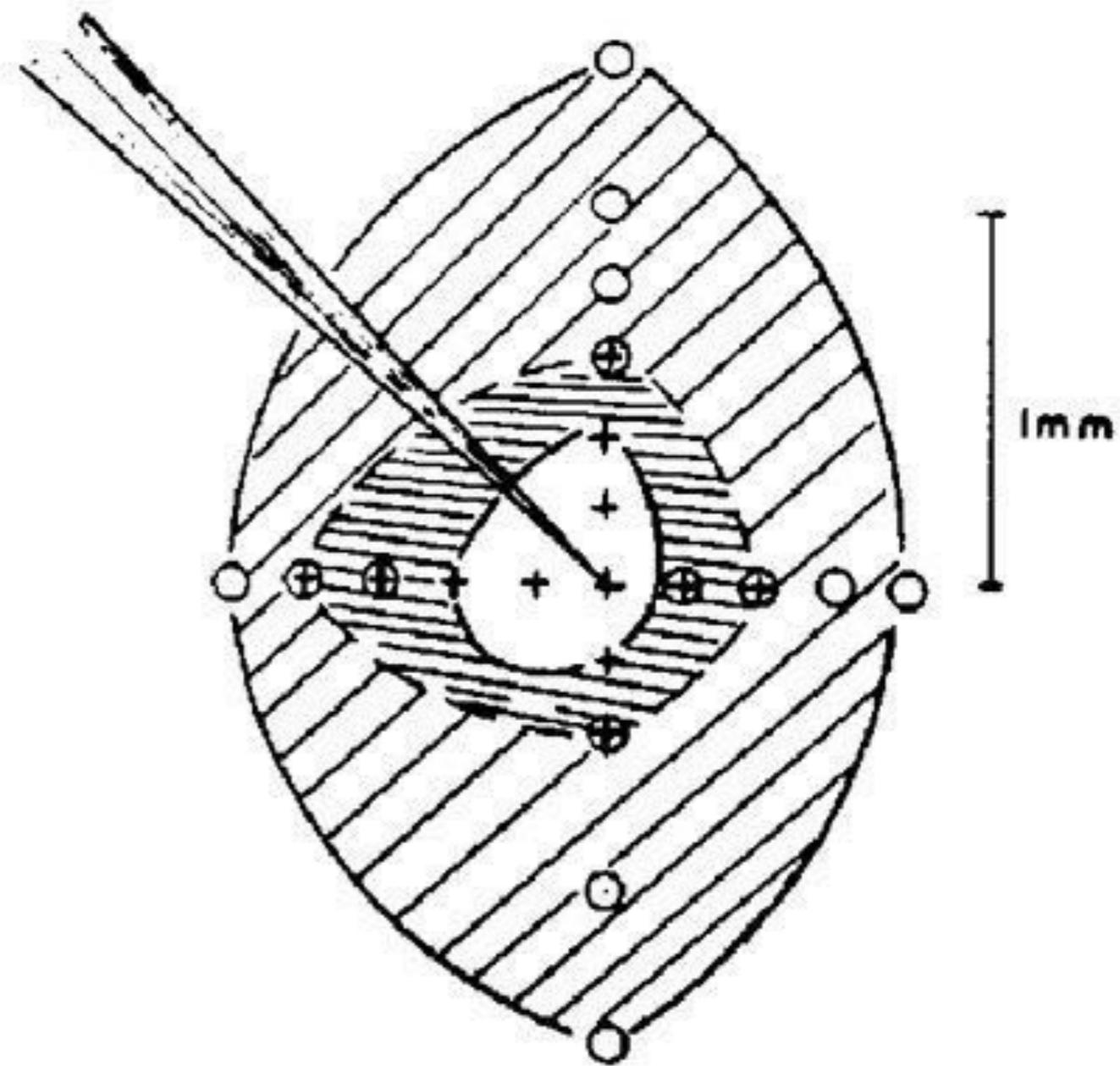
this photograph of Hartline in his laboratory is from long after the 1938 publication
 you can (if you - like me - have used several generations of Tectronics oscilloscopes)
 date the photograph from the equipment in the background

Hartline originally worked on *Limulus Polyphemus*, an ancient animal from the spider family that lives in the seas and regularly strands on the beaches near Boston
 Hartline would use a jigsaw to cut out the eye and draped the optic nerve over a silver electrode or would use a pipette microelectrode - it is how he discovered **lateral inhibition**





Stephen Kuffler
1913-1980



Stephen Kuffler

the physiological notion of the "receptive field" (RF) first occurs in the thirties:

"Responses can be obtained in a given optic nerve fiber only upon illumination of a certain restricted region of the retina, termed the receptive field of the fiber" (Hartline 1938)

Stephen Kuffler recorded the responses of retinal ganglion cells to spots of light in the cat (a mammal!) around 1950



NOTICE: the receptive fields may be considered to be necessarily ATOMIC because there is only a single degree of freedom in their activity - the spike frequency of the axon

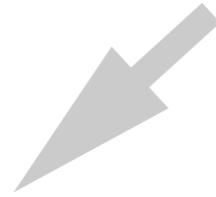
Brentano's edge point with two-way
plerosis is both "atomic" AND has "parts"

it is a **perceptive field** much like the
"receptive fields" from neurophysiology

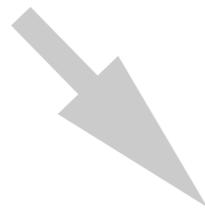
but: the notion of **receptive field** only
comes with Hartline in 1938 - Brentano's
psychology was ahead of brain science!

any white point somewhere here?

like here perhaps?



or here?



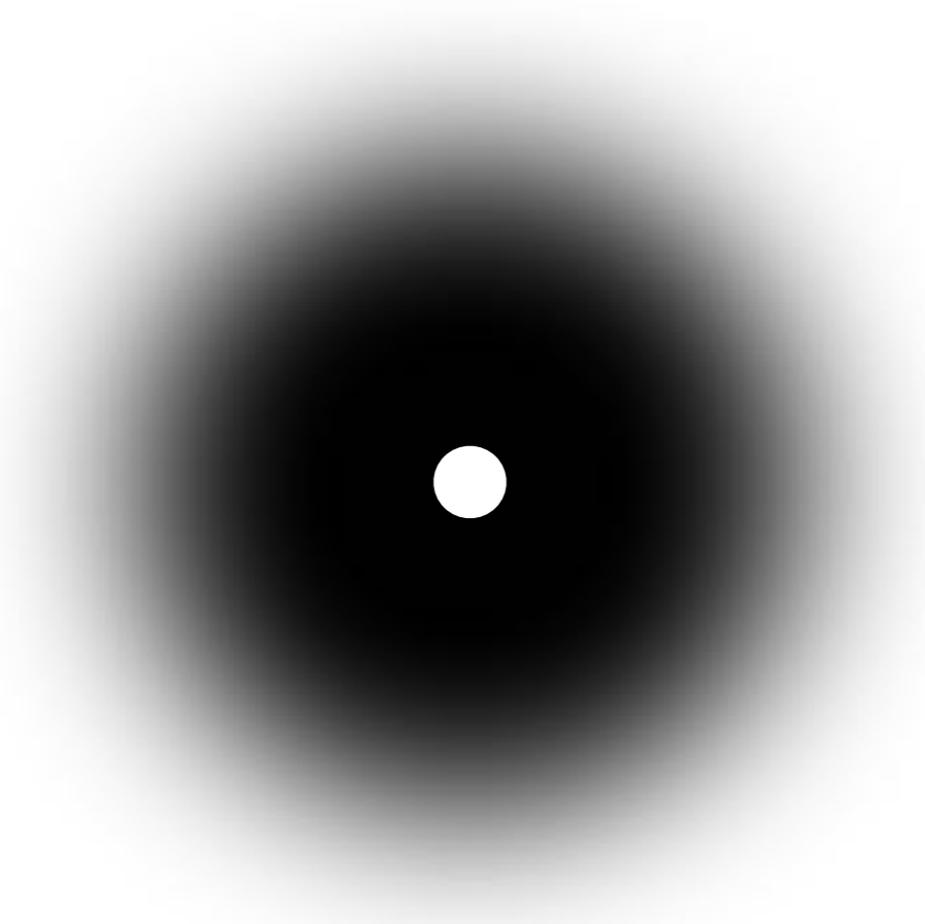
did you find some? then you must have found an infinity of them

you are probably a believer in Newton's absolute space
perhaps you think of "points" as Cartesian coordinates (2 numbers
defined in some frame) as many scientists do
but these are mere abstractions and lack reality

i discuss MATERIAL POINTS here, that is to say points that are
actually presented in your visual awareness

your visual awareness is CONCRETE ACTUALITY, anything else is
make-belief

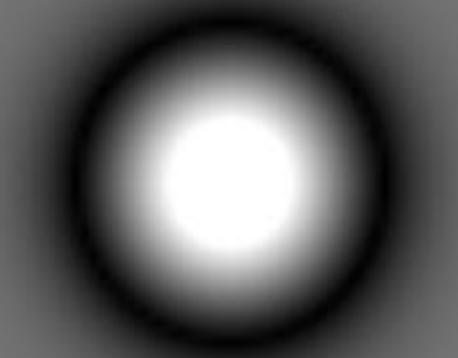
yes, but only if it has a background...



but now we have **two** points,
a large black one and a small white one

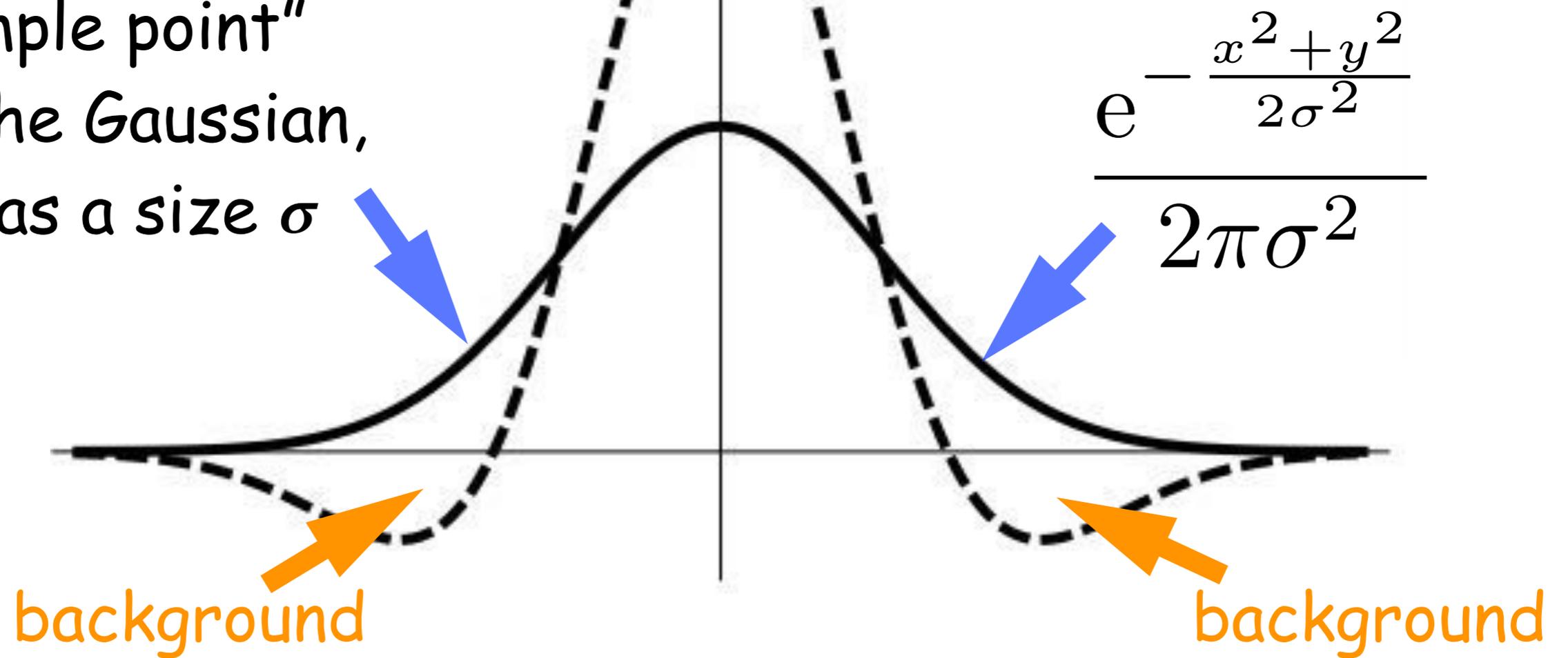


a point may carry its own background with it, it is a "center-surround" RF



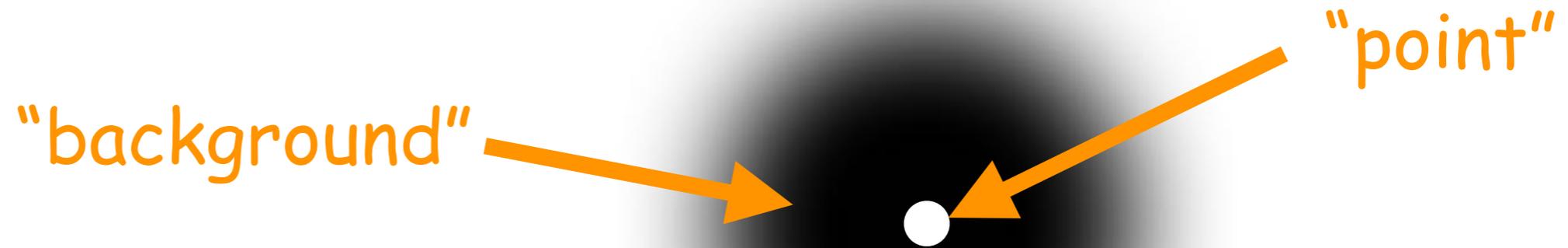
such a point is really the difference of two "simple" points of different sizes

the RF of a
"simple point"
is the Gaussian,
it has a size σ

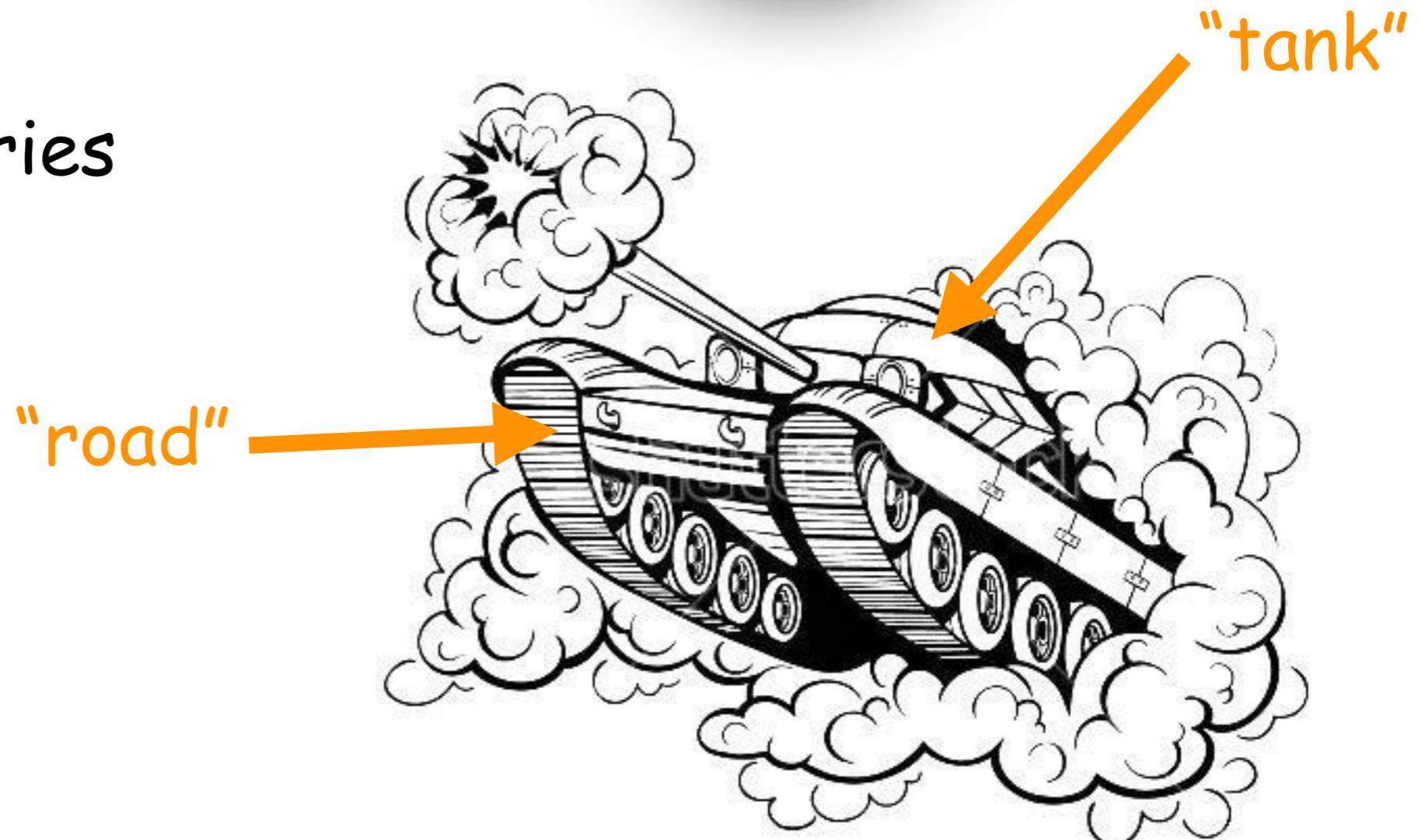


the point with background is visible, the point without background not - it does not exist!

"real" points are like receptive fields that carry their background with them ...



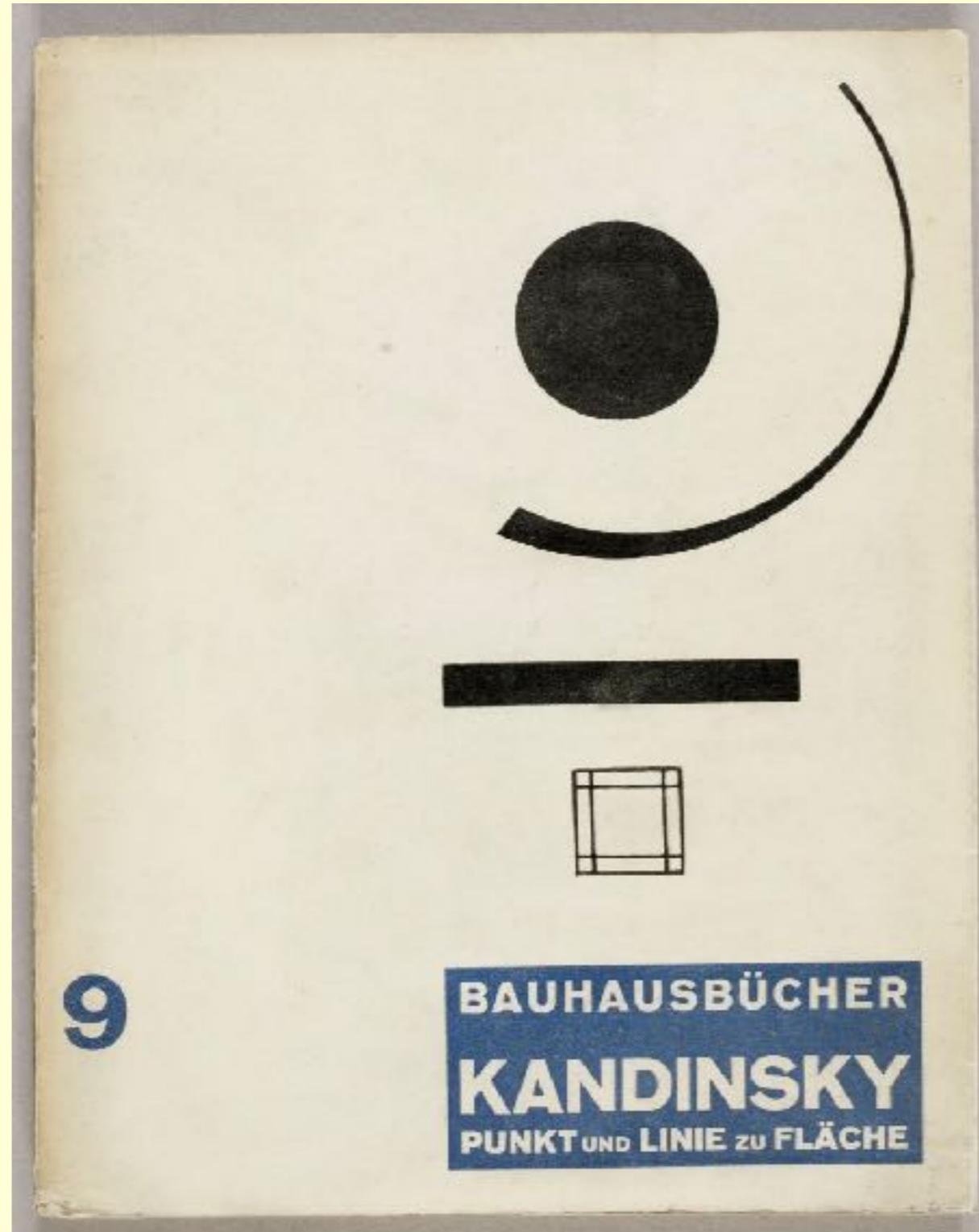
... like a tank carries its road along





Wassily Kandinsky
1866-1944

apart from looking at his paintings you may want to read his booklet "Punkt und Linie zu Fläche" of 1926



Kandinsky "a point may grow and fill the entire canvas..."

points may have any size

large points may be composed of small points

notice that Euclid never mentions that points should be small

Kandinsky "a point may grow and fill the entire canvas..."

points may have any size

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Kandinsky "a point may grow and fill the entire canvas..."

points may have any size

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notice that **Euclid never mentions that points should be small**

PLEASE NOTE:

in the following slides there were one or more videos that ran automatically and simultaneously in my KeyNote (confession: i use macs) version

since I had to convert to the simplest possible PDF, I had to down-sample the videos and insert them as arrays of image sequences - ON THE NEXT PAGE(S)

this is perhaps unfortunate, but I can do little about it

VIDEO

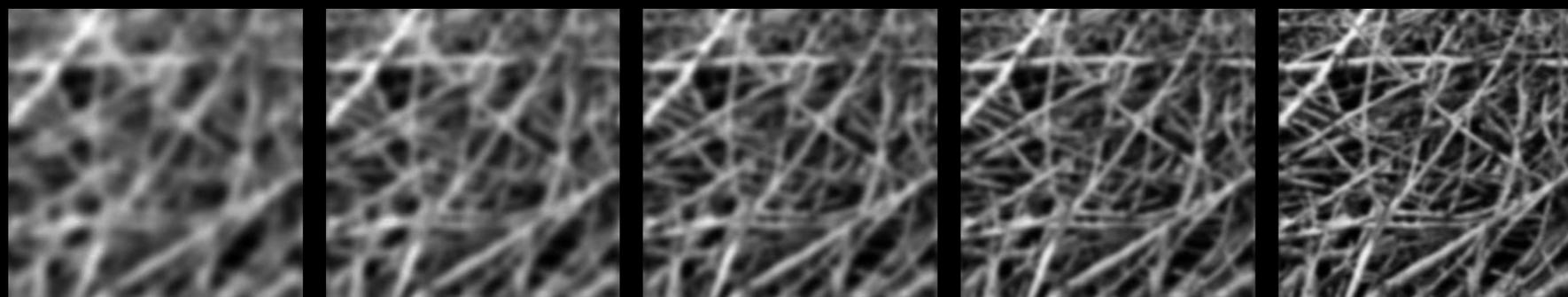
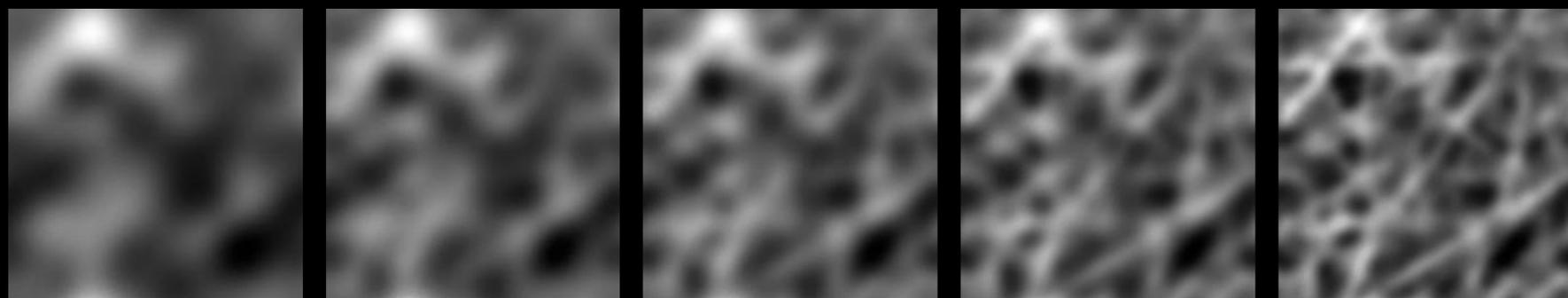
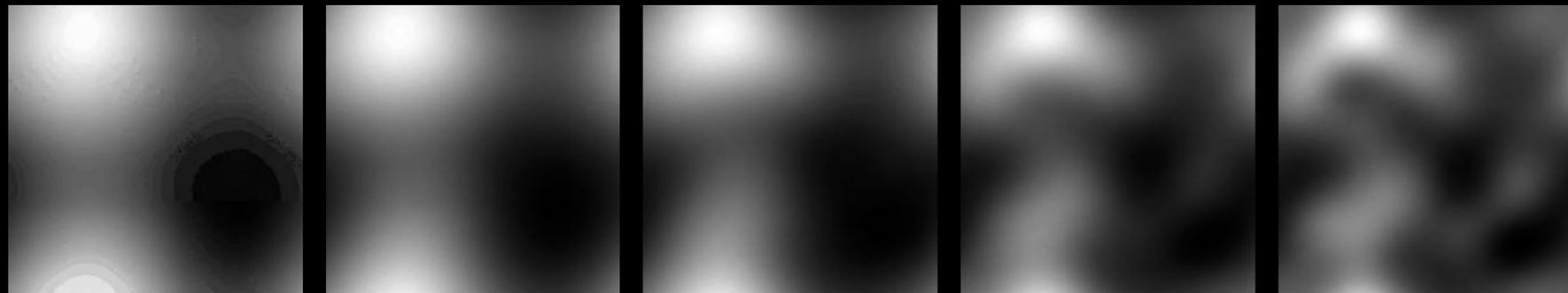
any image as "seen by" various point-sizes will look very differently according to size

you only see "the trees" or "the forest", but never both simultaneously!

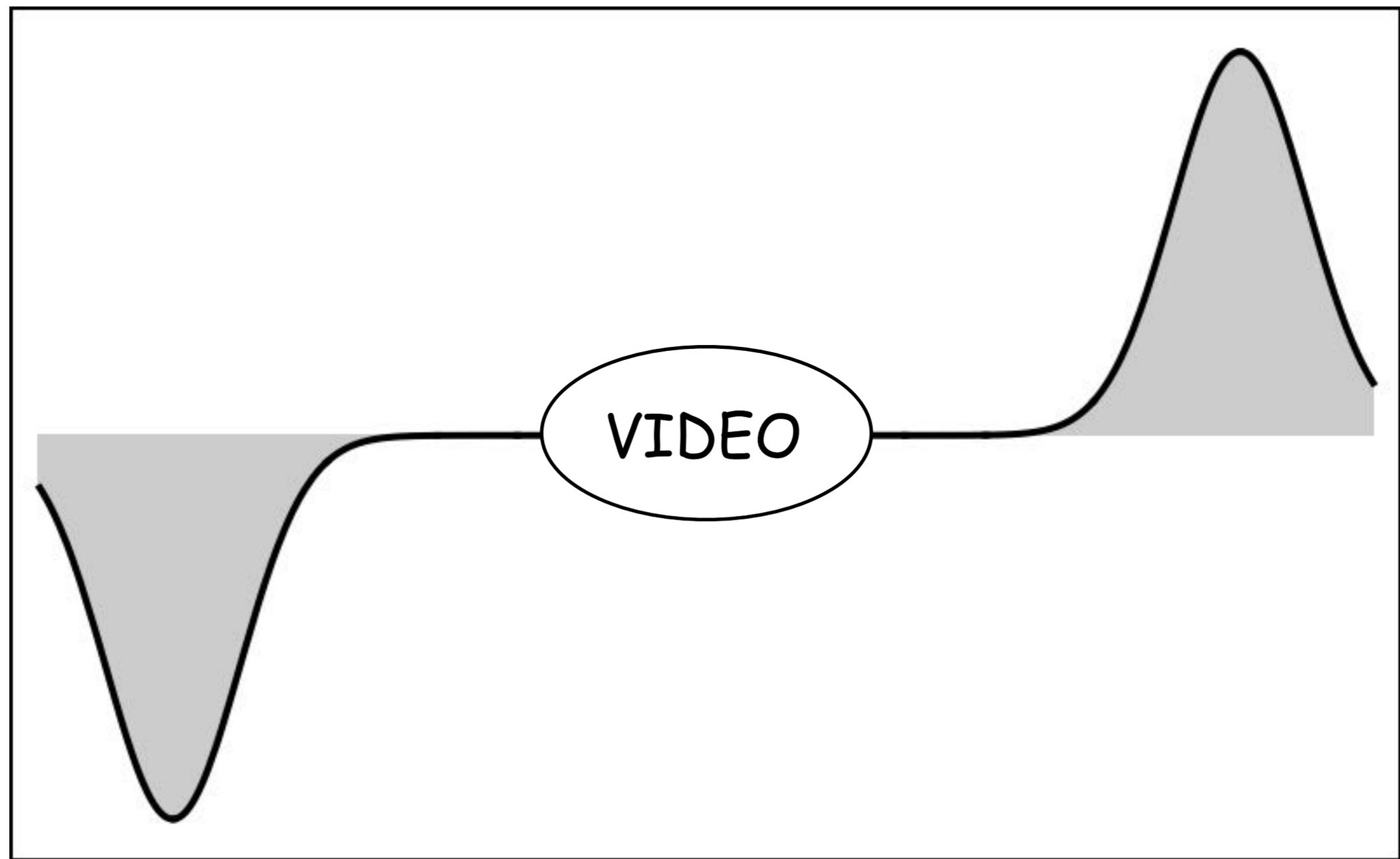
all such images are equally valid



the trees may be more important than the forest, or vice versa

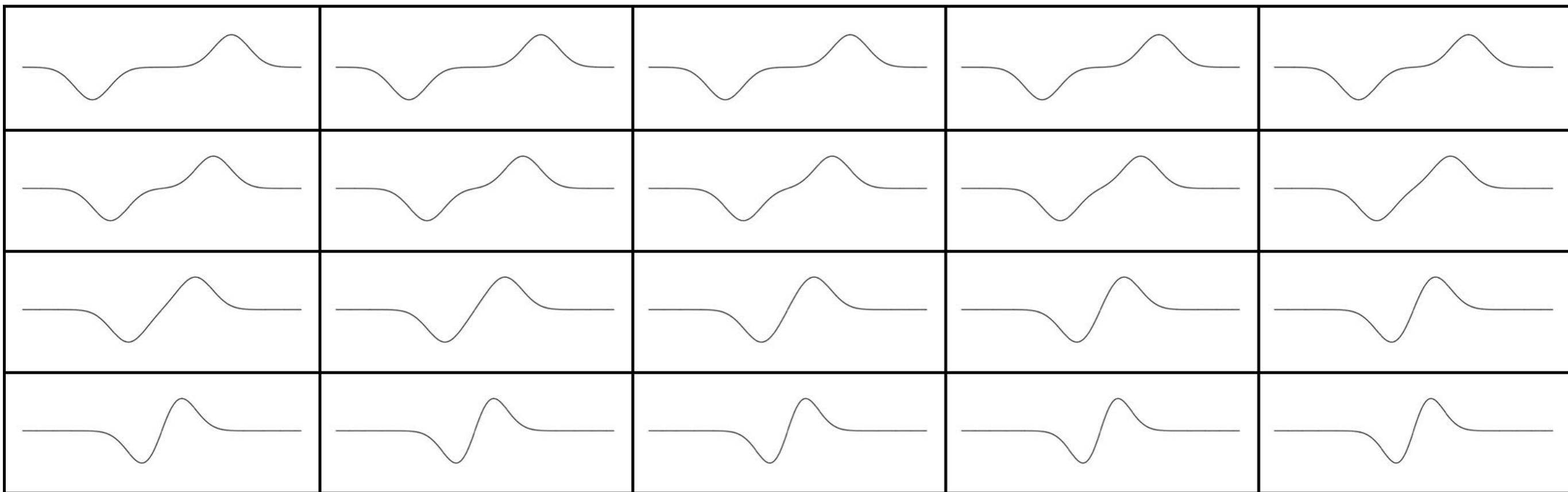


mosaic
of
video

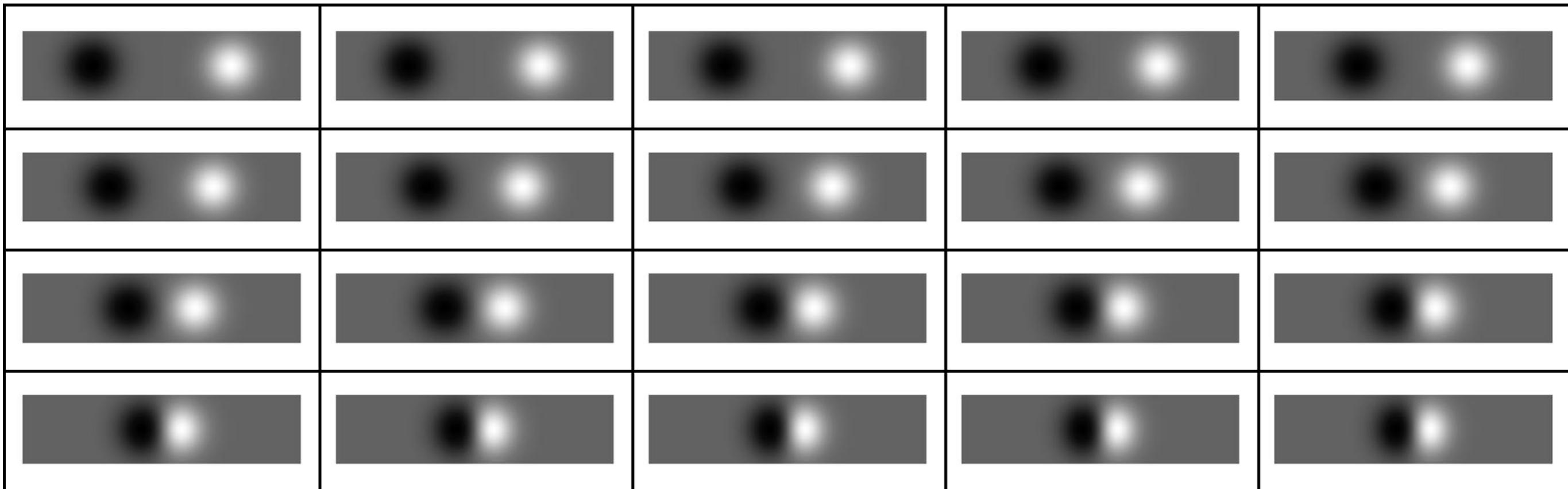


$$\frac{x e^{-\frac{x^2 + y^2}{2\sigma^2}}}{2\pi\sigma^4}$$

"the difference of 2 coincident(!) points" neatly captures the idea of Brentano's plerosis for an edge



mosaic of videos



notice that

$$\lim_{d \downarrow 0} \frac{\frac{e^{-\frac{\left(x + \frac{d}{2}\right)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}} - \frac{e^{-\frac{\left(x - \frac{d}{2}\right)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}}}{d} = \frac{\partial}{\partial x} \left(\frac{e^{-\frac{\left(x - \frac{d}{2}\right)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}} \right) = -\frac{x e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^3}}$$

thus the edge finder profile is just the difference of two "coincident" (but ordered!) point profiles



Élie Joseph Cartan

1869

- 1951

Cartan ... went to the family smithy and helped his father and brother to blow the blacksmith's bellows ...

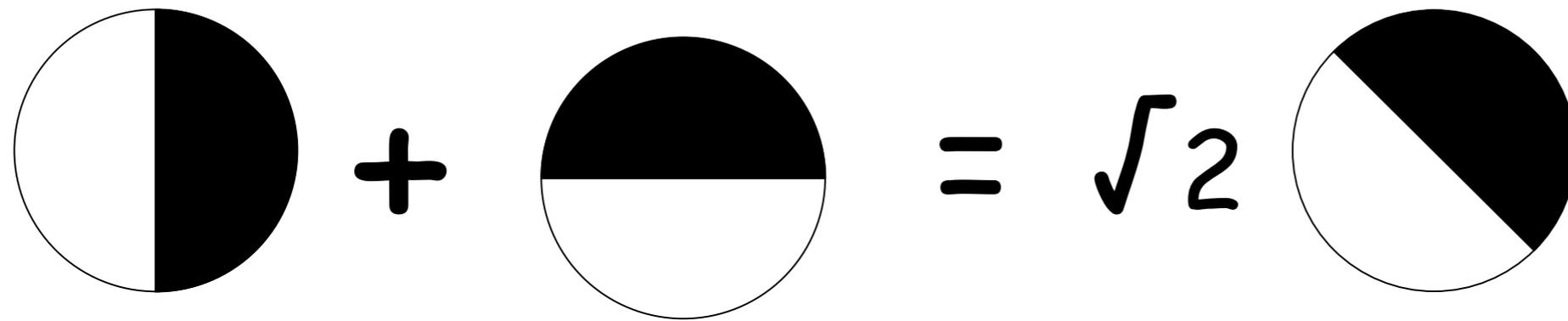
(yet another scientist with his feet on the ground!)

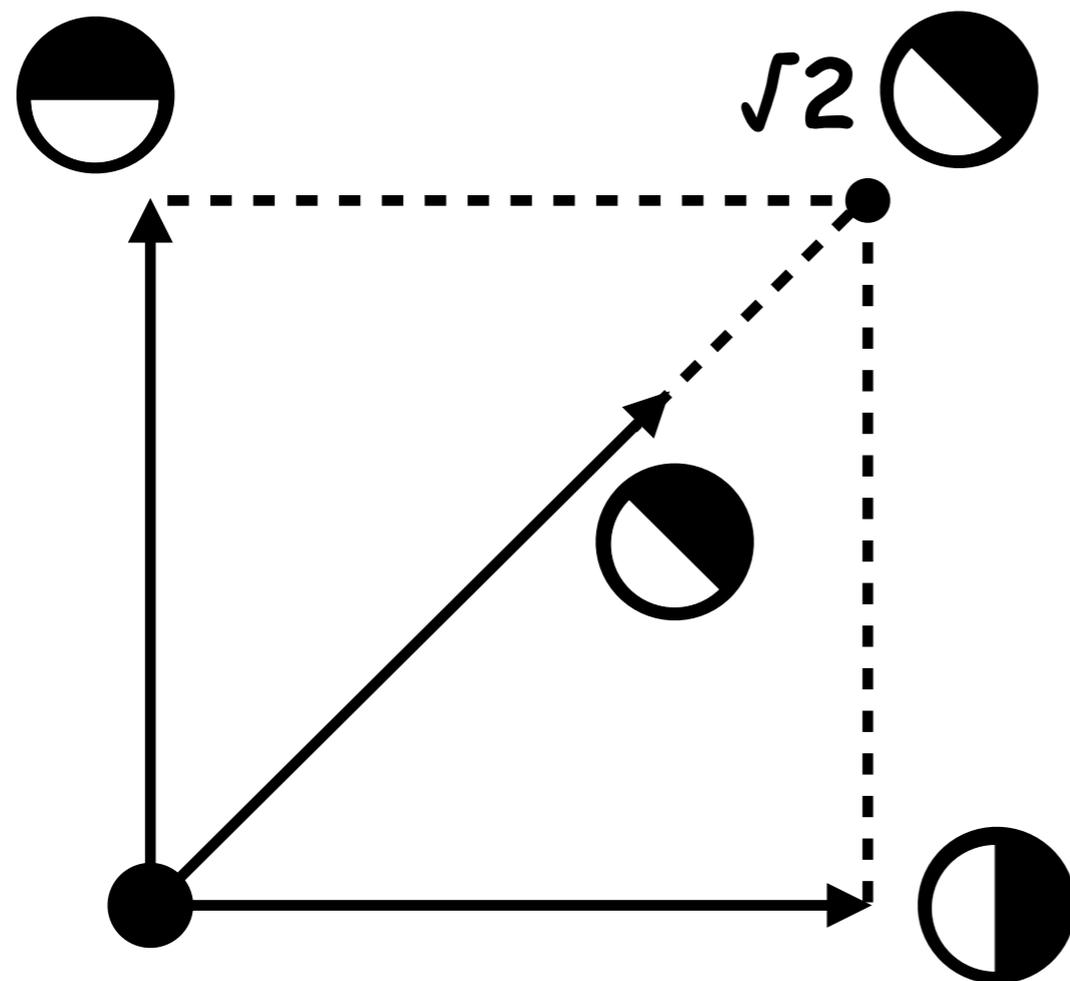
formally, it is the "derivative of a point" (introduced in formal geometry by Elie Cartan in the 1920's)

in image processing it was introduced as an "edge detector"

in brain science the corresponding receptive field is often called "edge detector" too - its activity represents local "edginess"

typically there will be some edginess EVERYWHERE
if you want edges, you need to pick & choose, for instance,
by way of a threshold


$$\text{Circle (left split)} + \text{Circle (top split)} = \sqrt{2} \text{ Circle (diagonal split)}$$



edge finders combine
like vectors

formally, they **ARE**
vectors!

this is nothing special, of course

since edge finders are derivatives of a point they are tangent vectors of the visual field

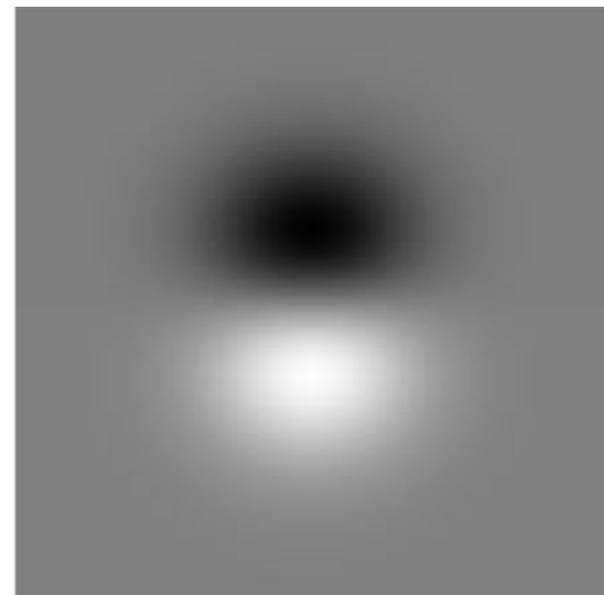
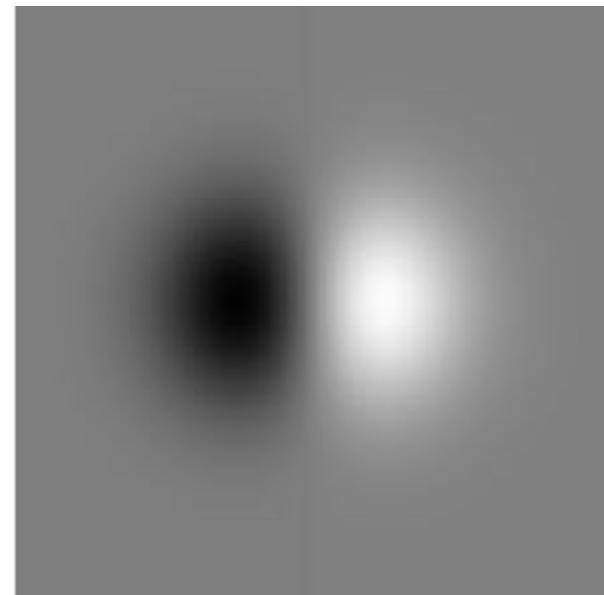
in the next slides i show a half turn (180°) in 10° steps

the formula i use is (with $r^2 = x^2 + y^2$)

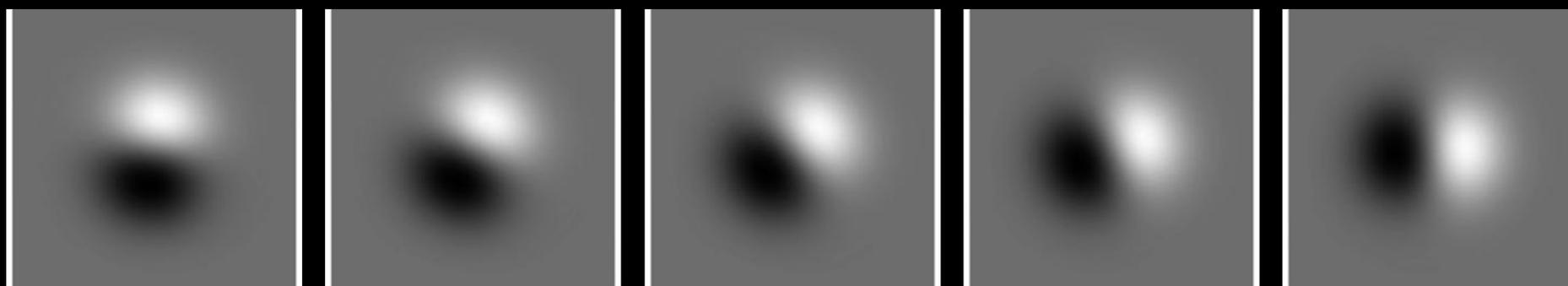
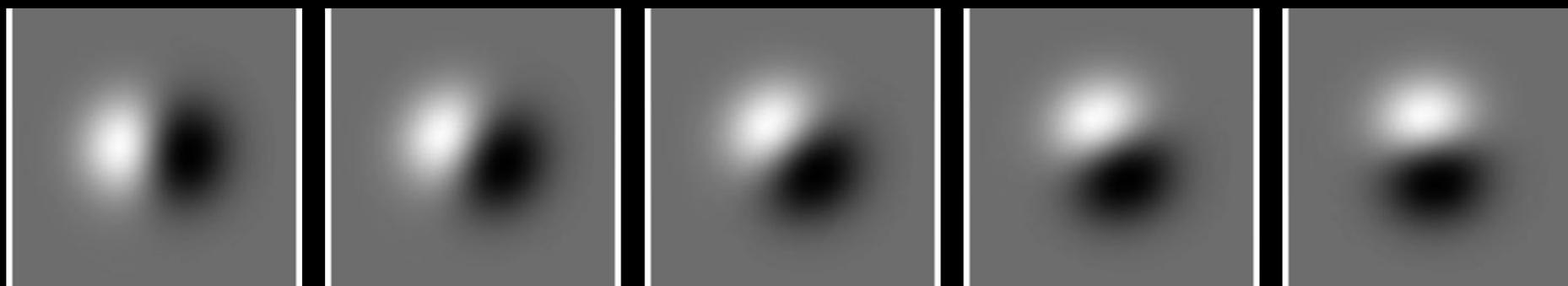
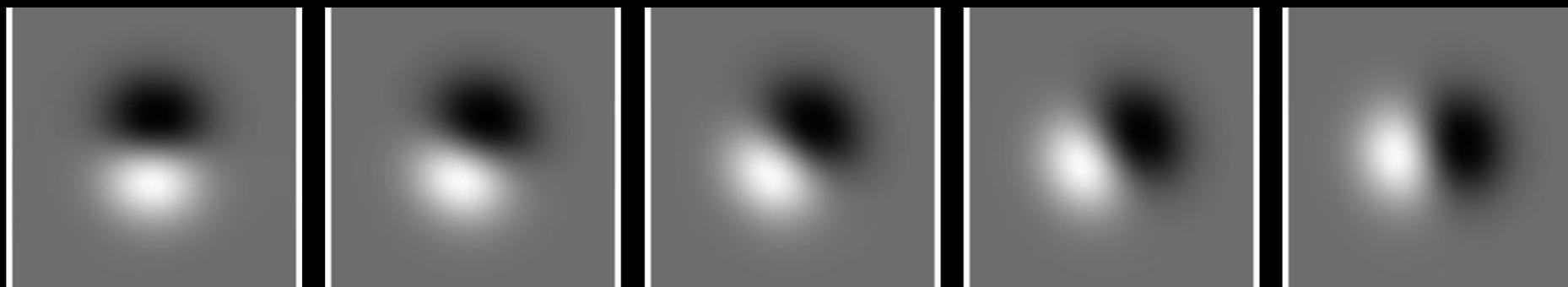
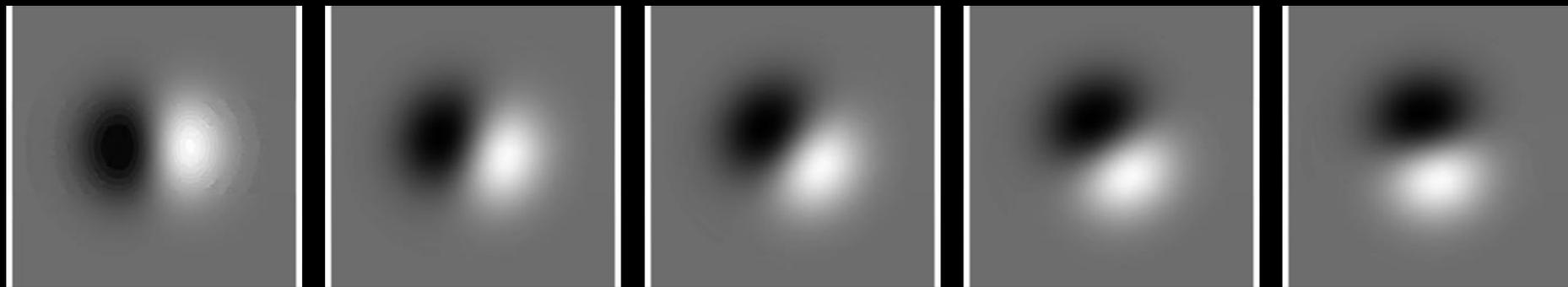
$$\partial_\varphi \left(\frac{e^{-\frac{r^2}{2\sigma^2}}}{2\pi\sigma^2} \right) = \cos \varphi \partial_0 \left(\frac{e^{-\frac{r^2}{2\sigma^2}}}{2\pi\sigma^2} \right) + \sin \varphi \partial_{\frac{\pi}{2}} \left(\frac{e^{-\frac{r^2}{2\sigma^2}}}{2\pi\sigma^2} \right)$$

where ∂_φ stands for spatial differentiation in the direction φ

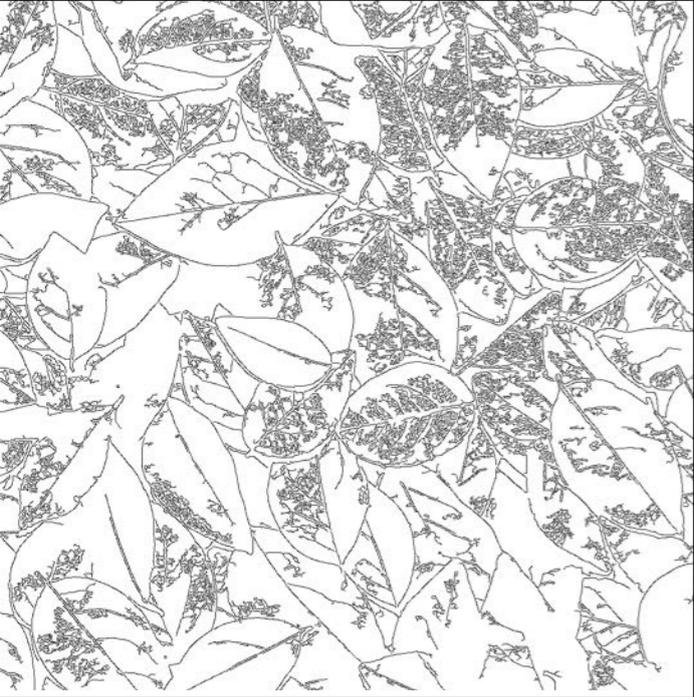
VIDEO



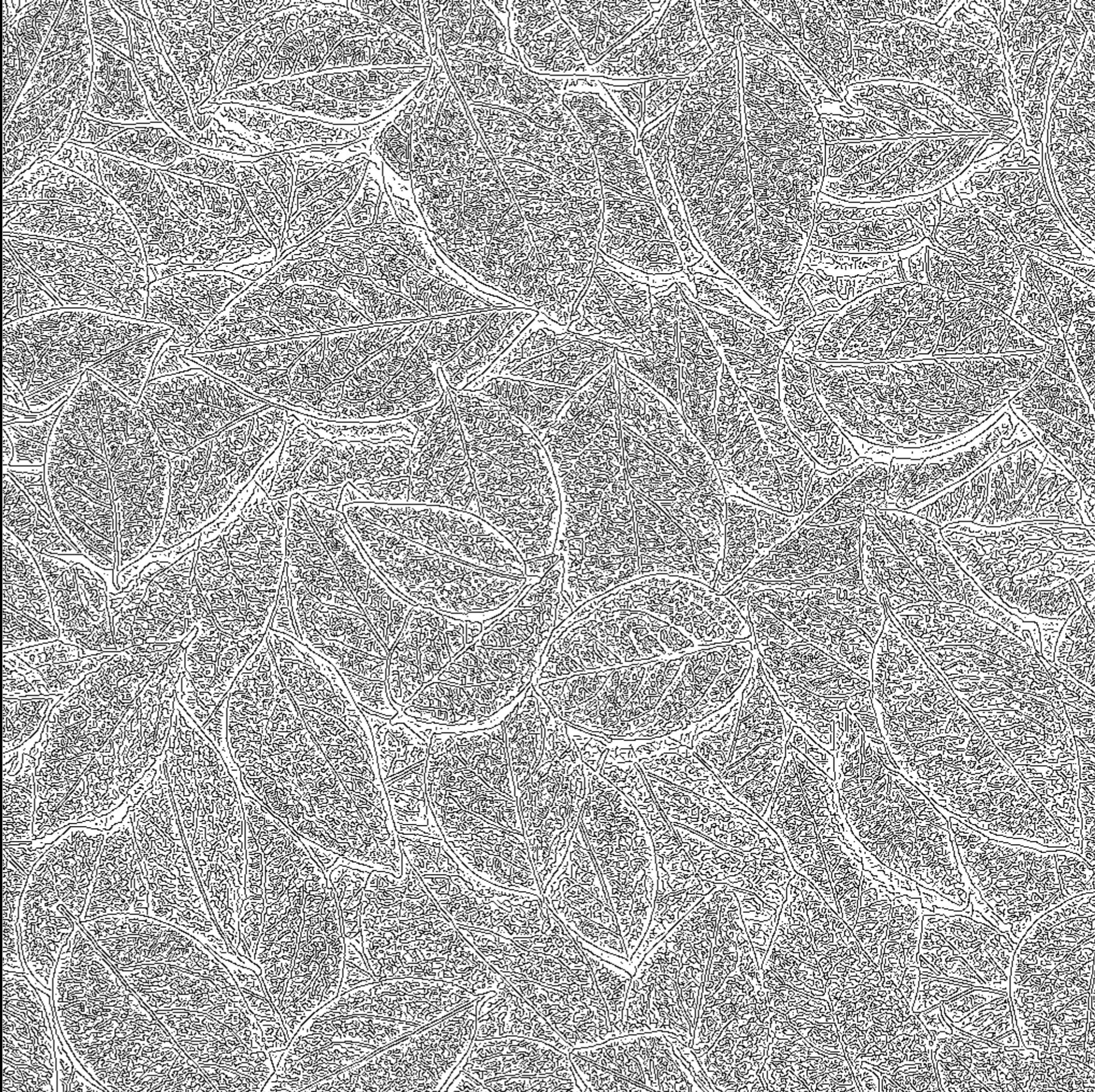
combination of
two makes all!

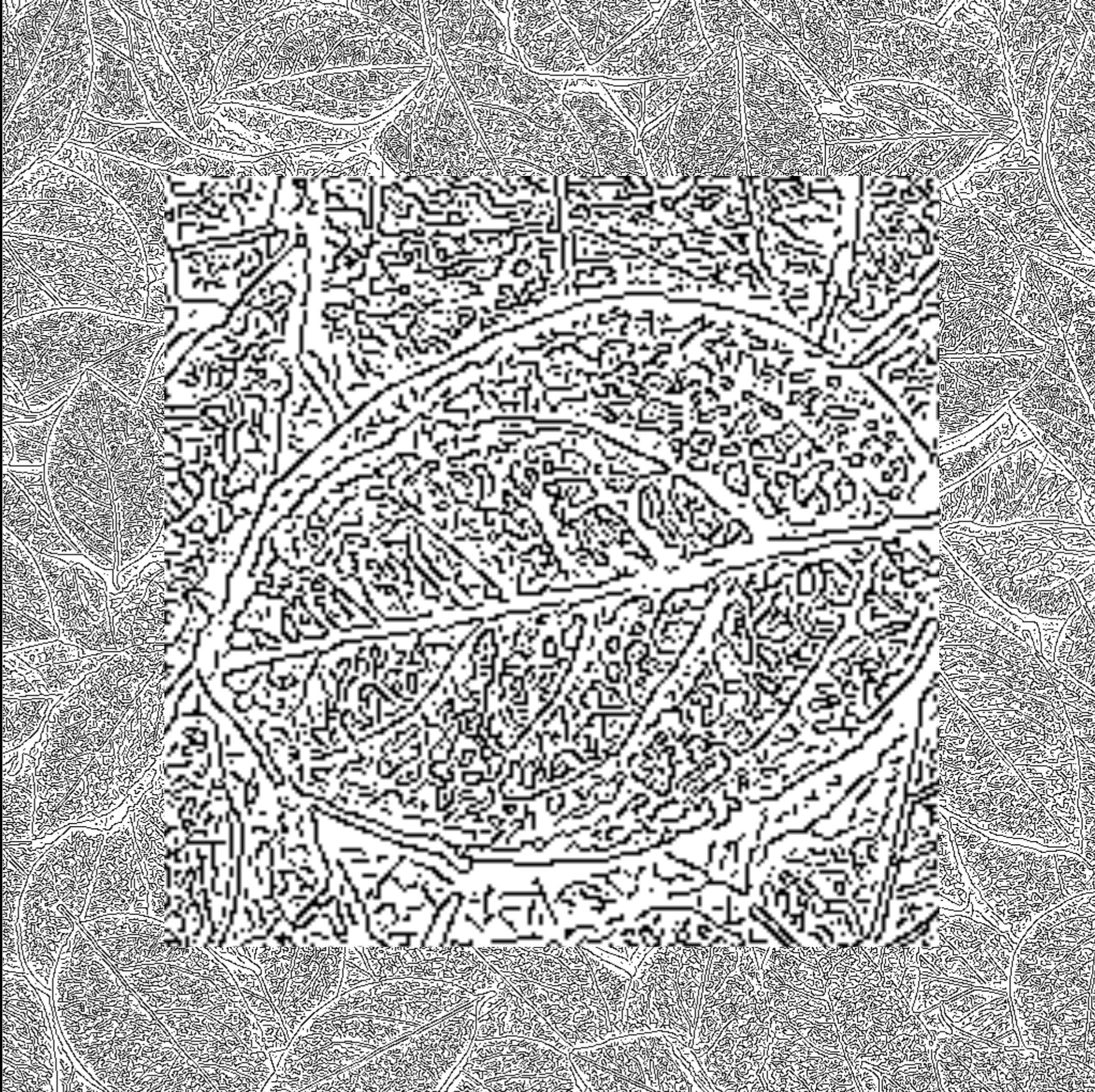
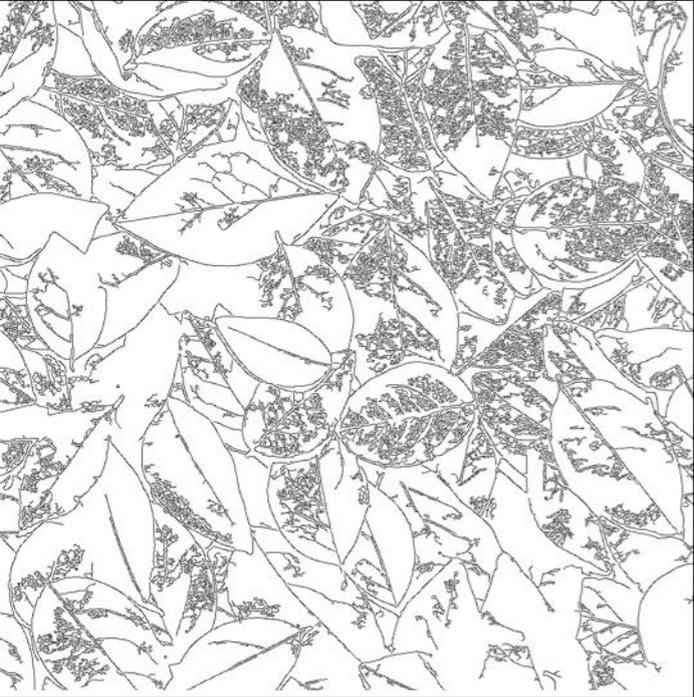


mosaic
of
video



picking
edges

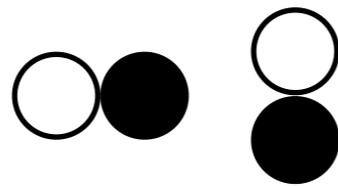




picking
edges

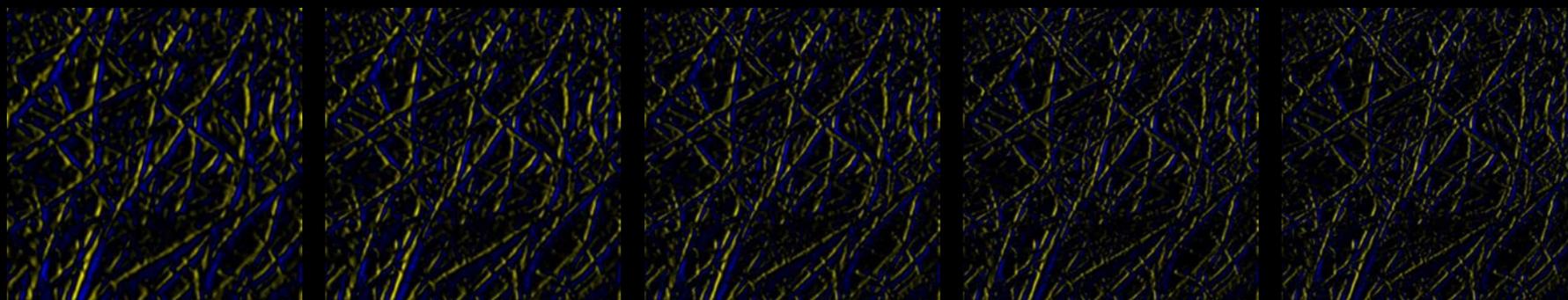
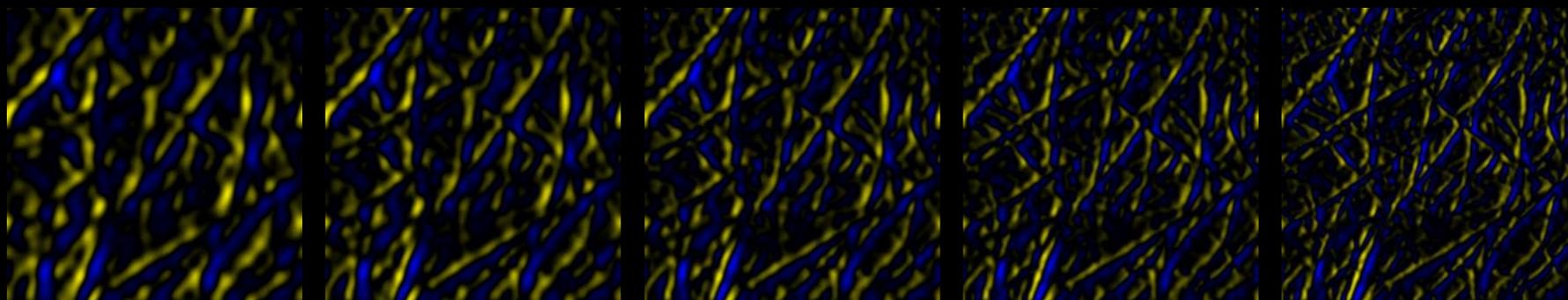
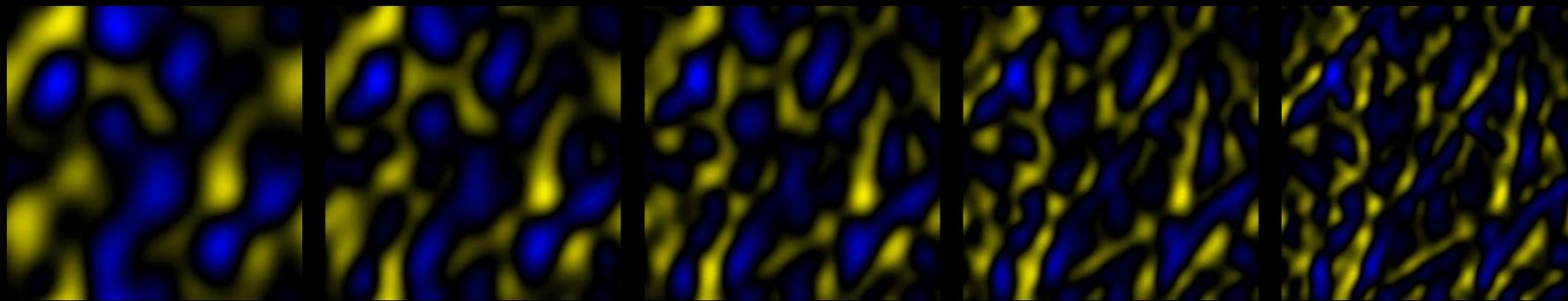
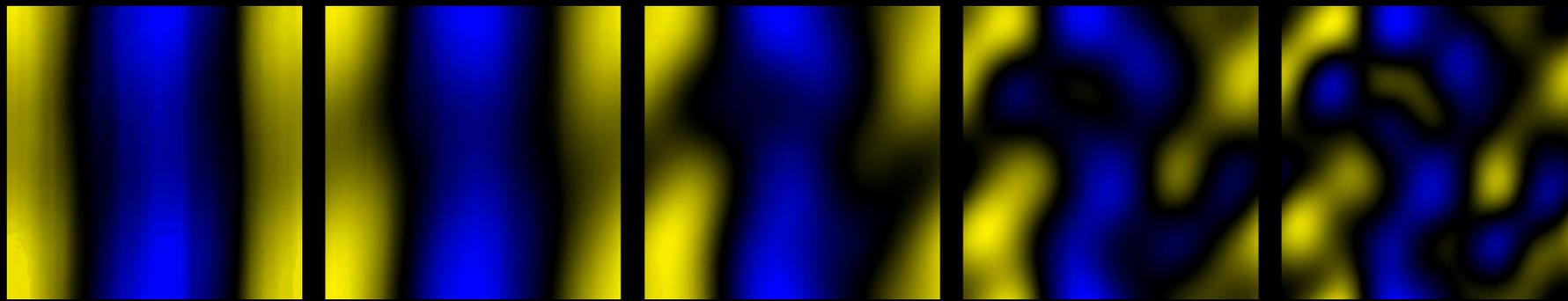
VIDEO

VIDEO

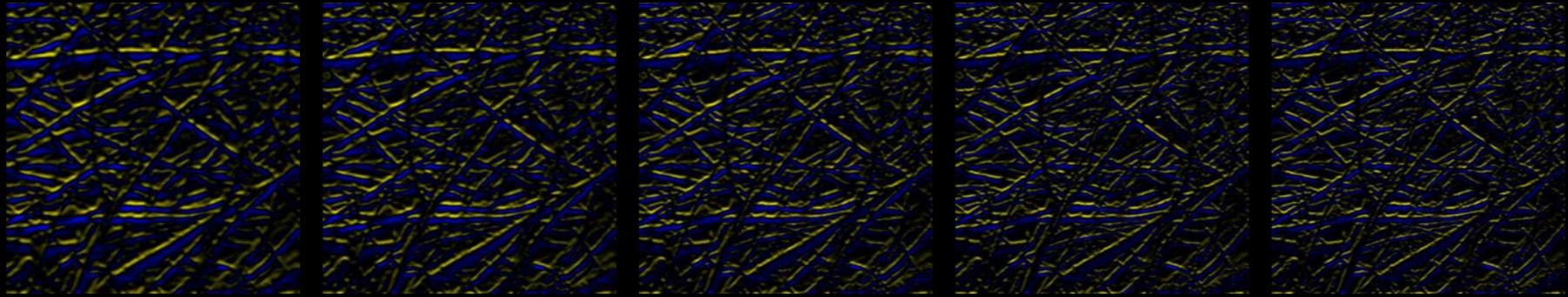
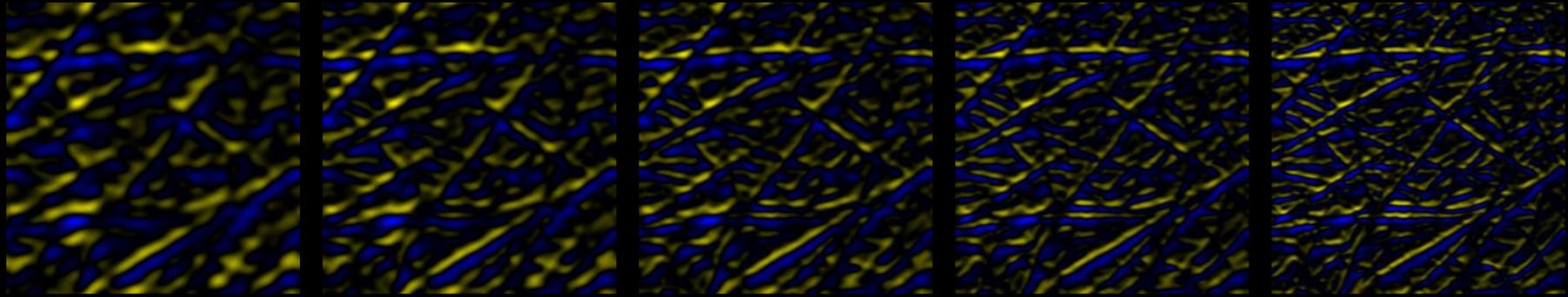
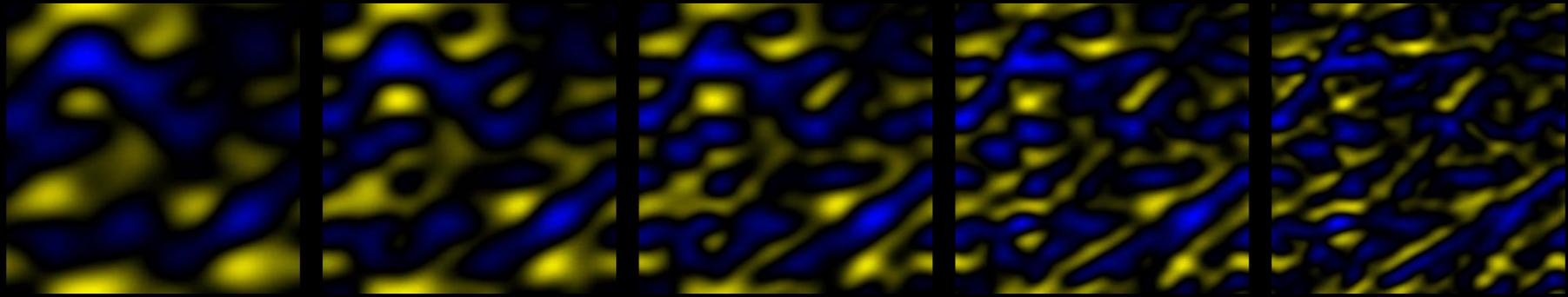
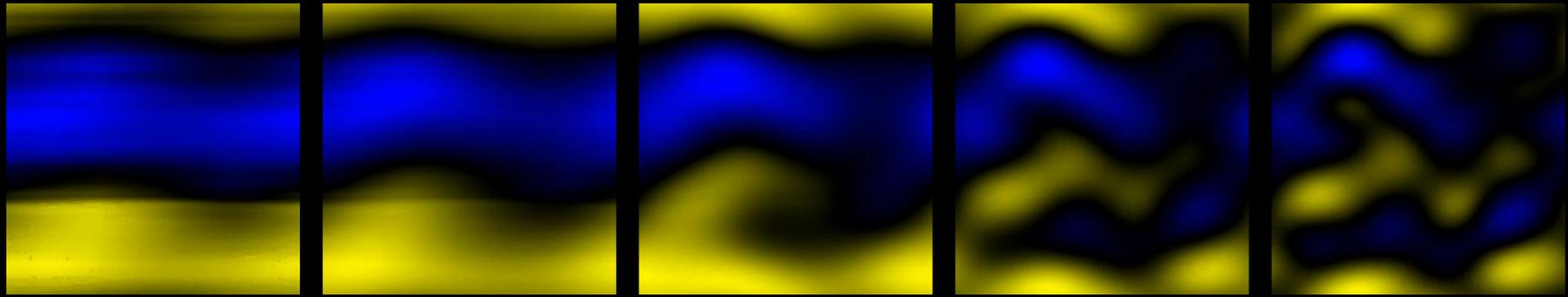


for "edges" we have at least two directions to consider, for each direction the edge can be with that direction or against it





mosaic
of
video I

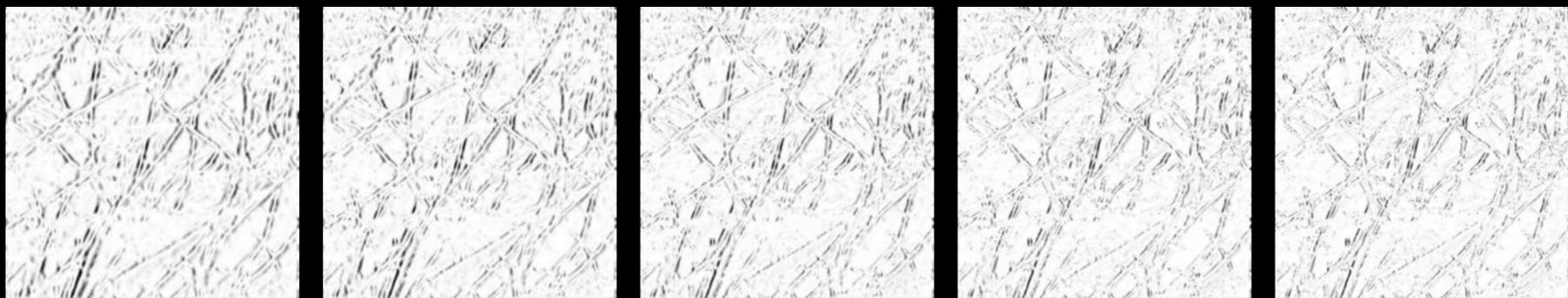
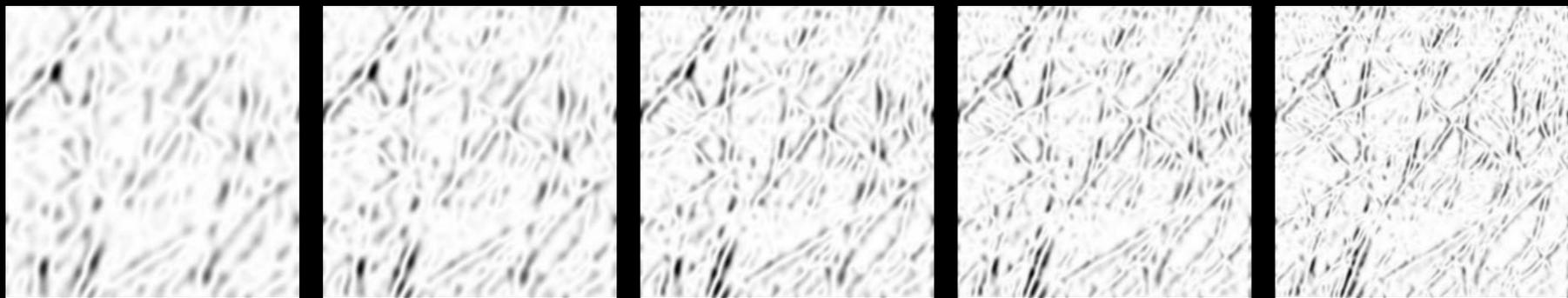
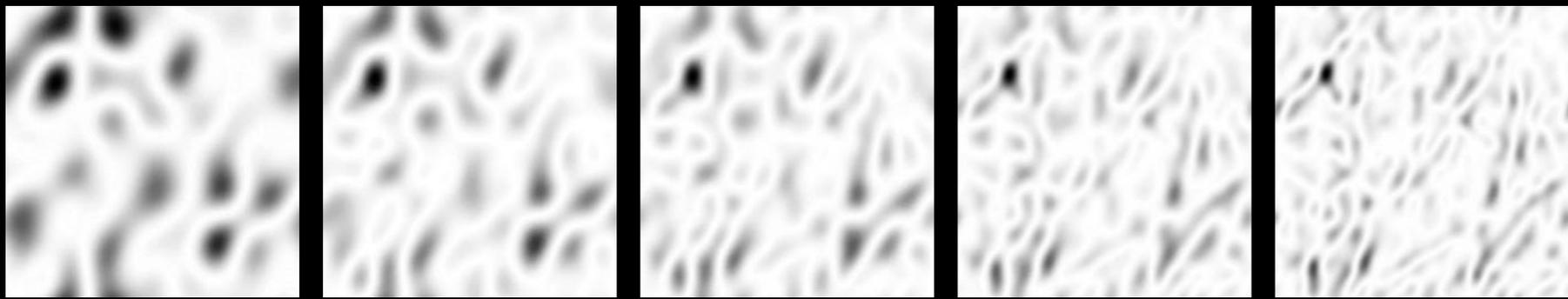
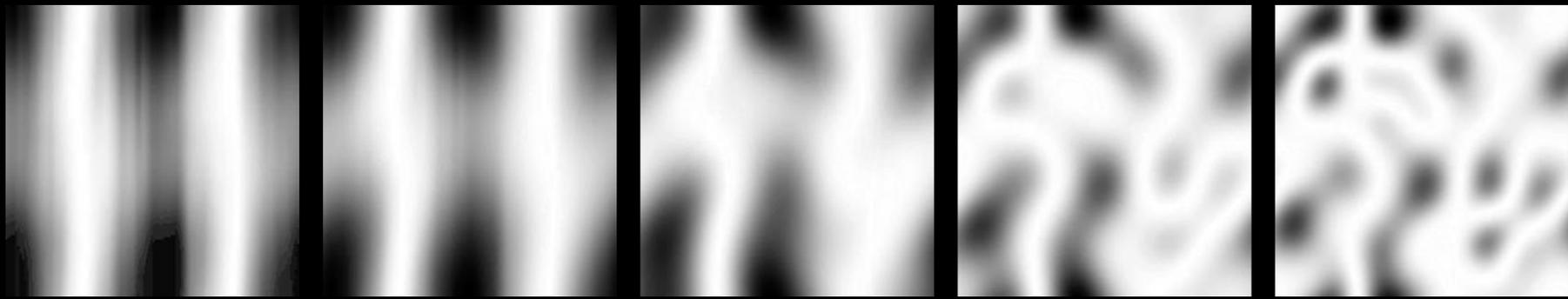


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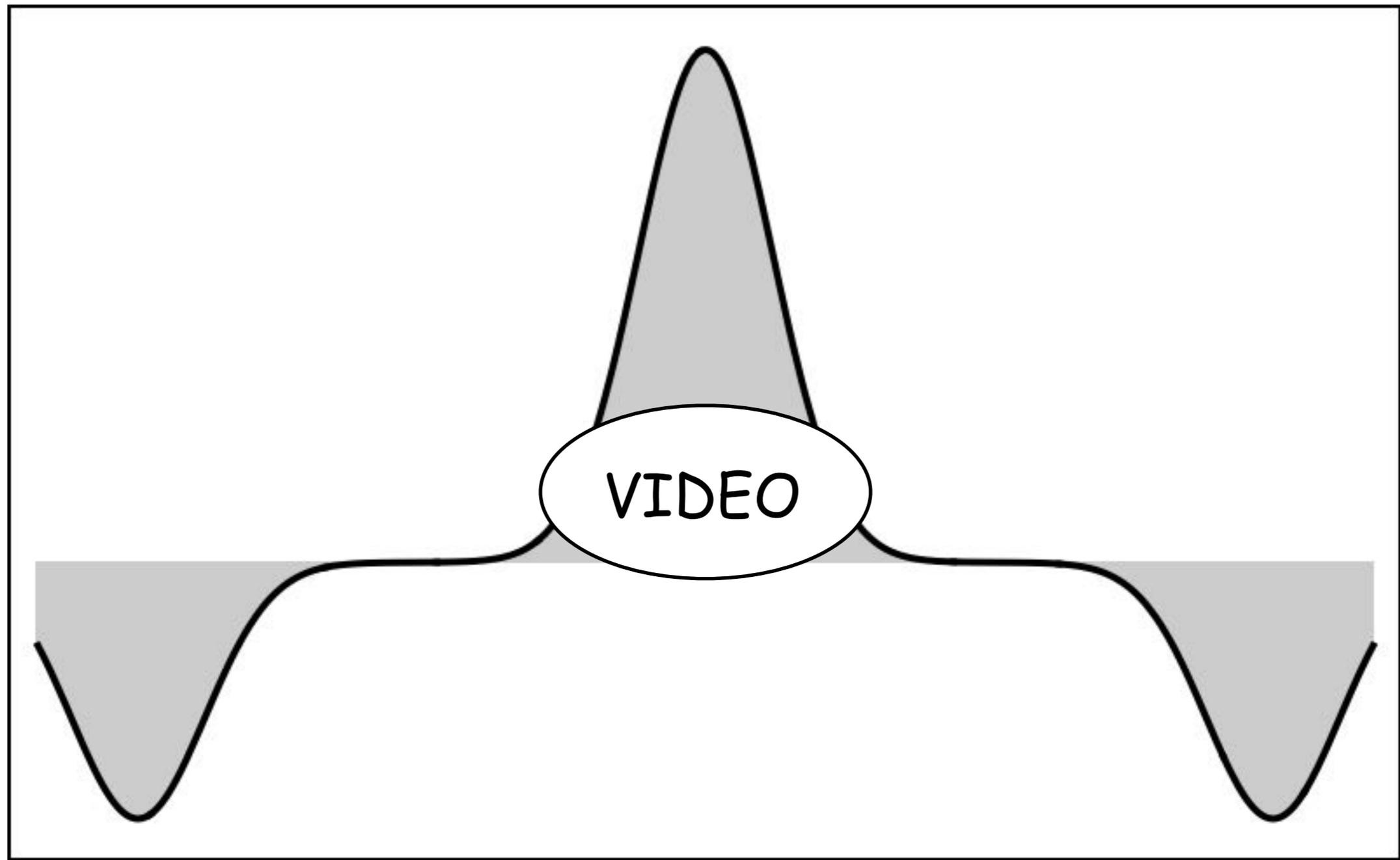
VIDEO

here we have all edge contributions combined in a single "edginess" measure



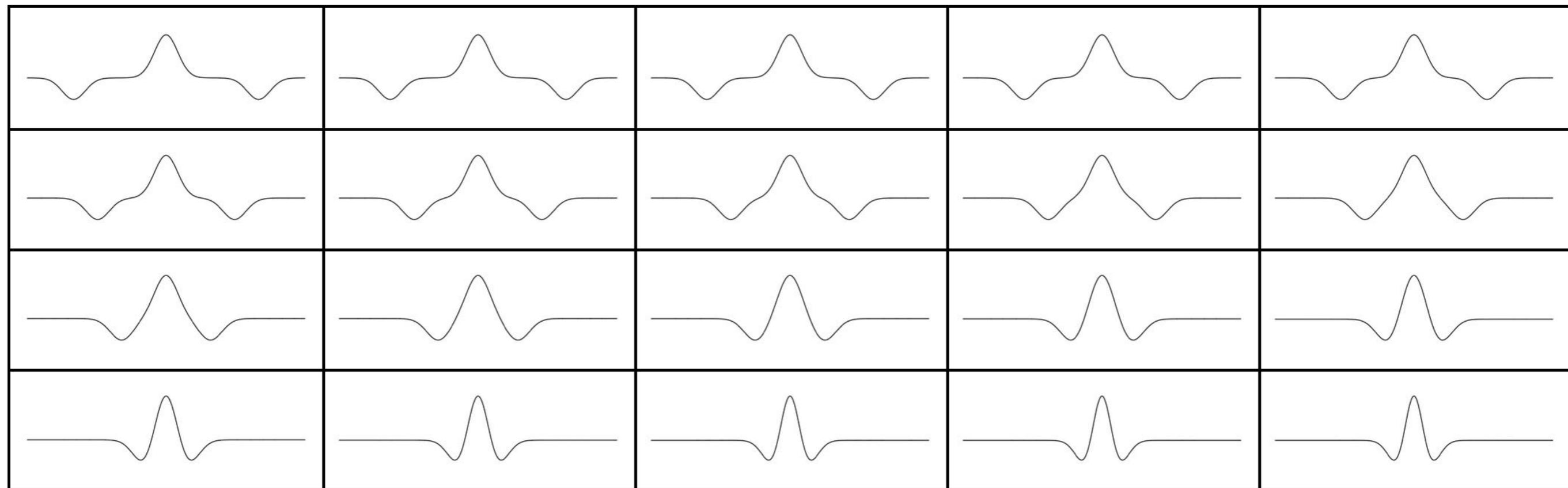


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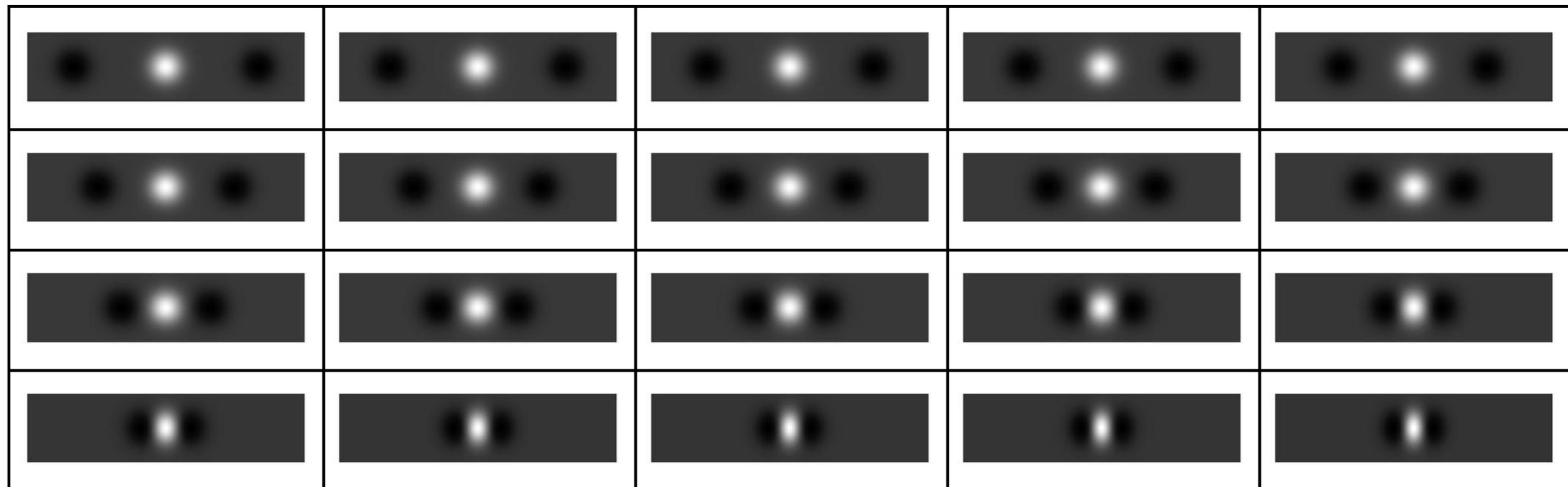


$$\frac{(x^2 - \sigma^2) e^{-\frac{x^2 + y^2}{2\sigma^2}}}{2\pi\sigma^6}$$

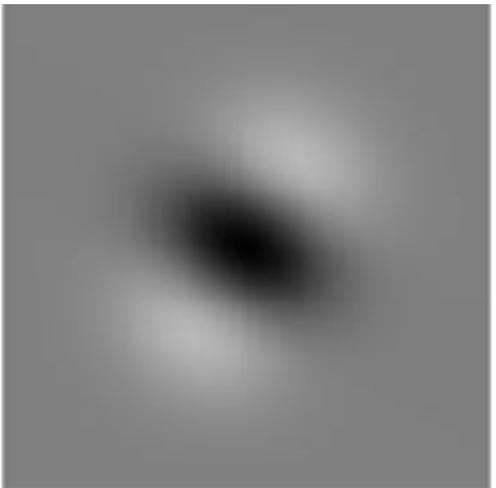
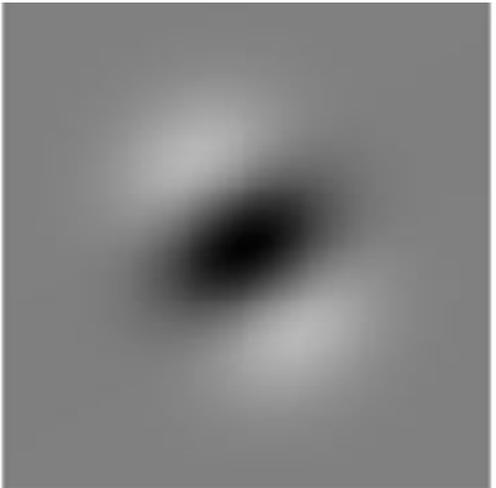
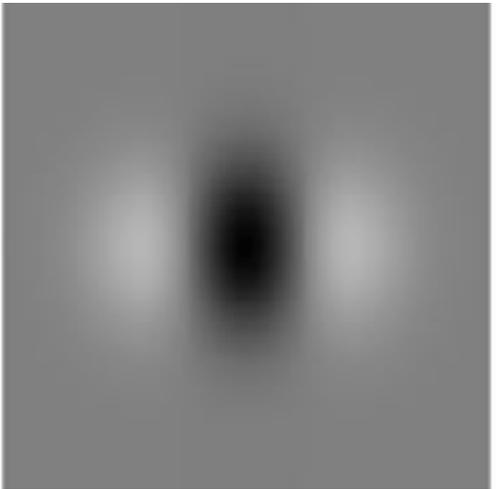
"the difference of 3 coincident(!) points" again captures Brentano's plerosis for a "linelet"

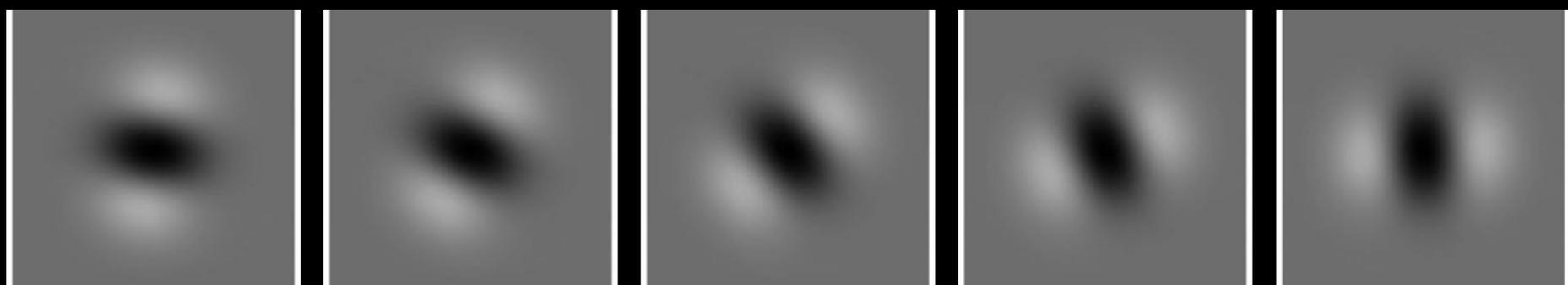
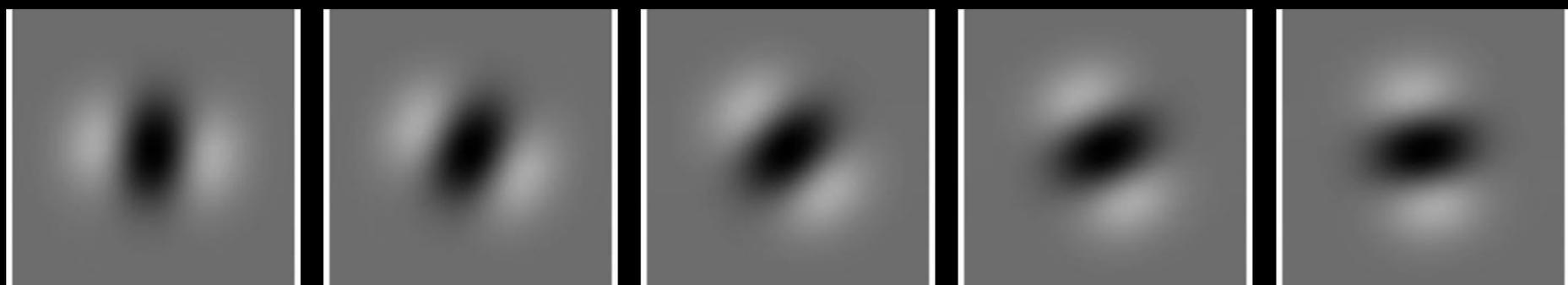
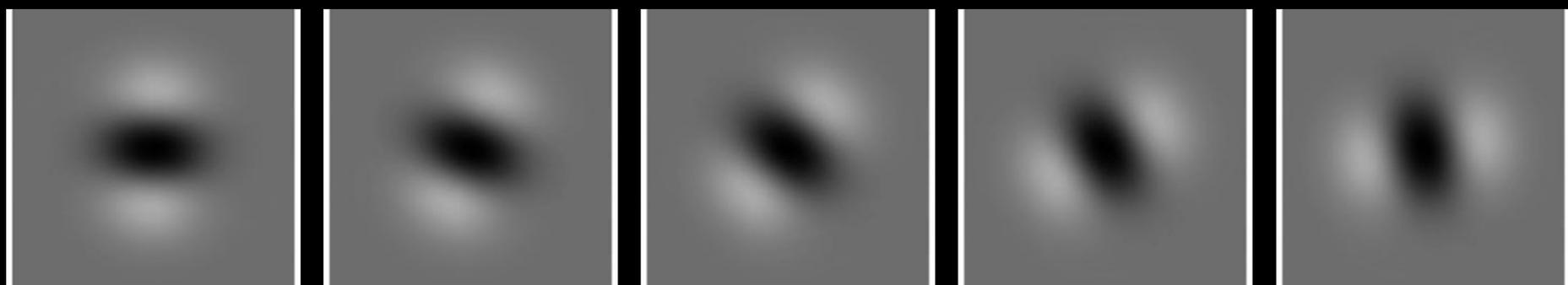
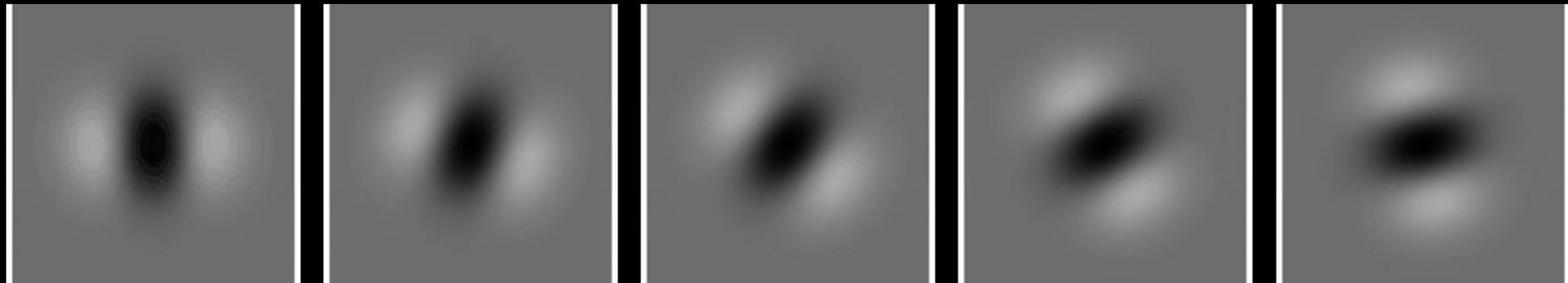


mosaic of videos



VIDEO





mosaic
of
video



David Hubel (1926-2013) & Torsten Wiesel (1924) [NP1981]

EVOLUTION OF IDEAS ON THE PRIMARY VISUAL CORTEX, 1955-1978: A BIASED HISTORICAL ACCOUNT

Nobel lecture, 8 December 1981

by
DAVID H. HUBEL

Harvard Medical School, Department of Neurobiology,
Boston, Massachusetts, U.S.A.

INTRODUCTION

In the early spring of 1958 I drove over to Baltimore from Washington, D.C., and in a cafeteria at Johns Hopkins Hospital met Stephen Kuffler and Torsten Wiesel, for a discussion that was more momentous for Torsten's and my future than either of us could have possibly imagined.

formally, it is the "2nd-order derivative of a point"

in image processing it was introduced as a "ridge detector"

in brain science the corresponding receptive field is often called "line detector" after Hubel & Wiesel - its activity represents local "lineness"

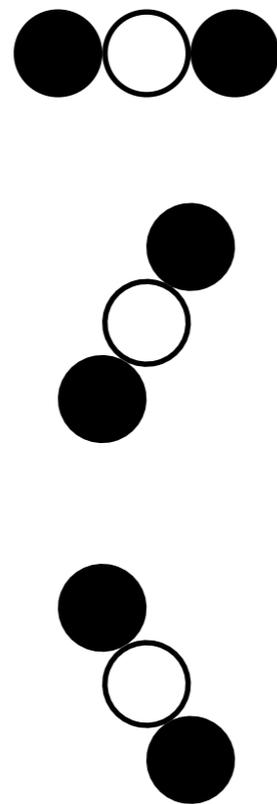
typically there will be some lineness EVERYWHERE
if you want lines, you again need to pick & choose, for instance, by way of a threshold

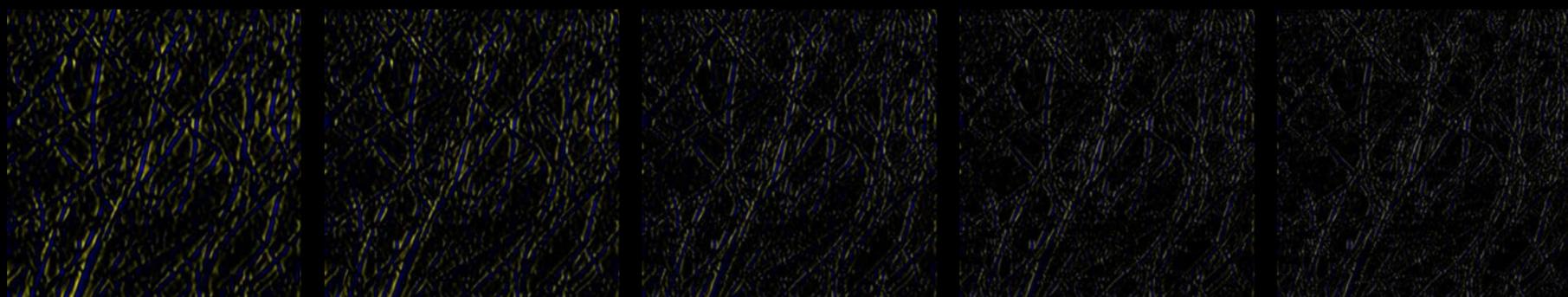
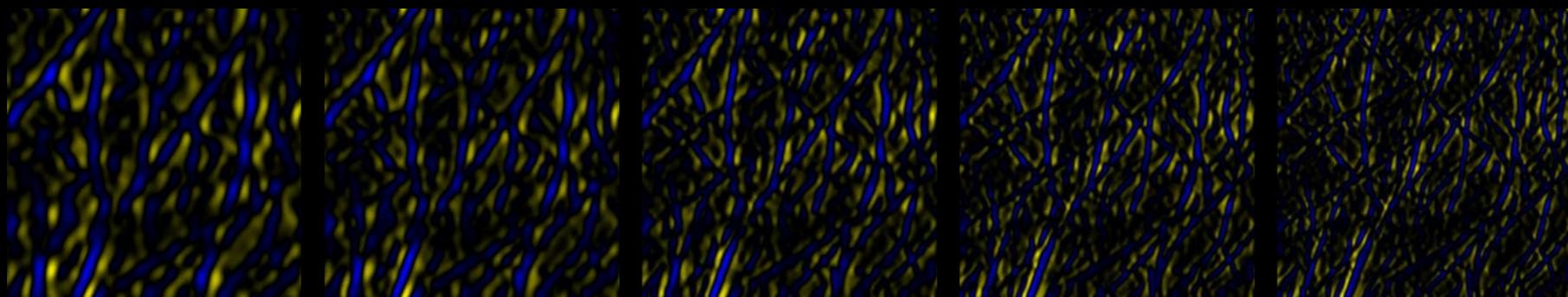
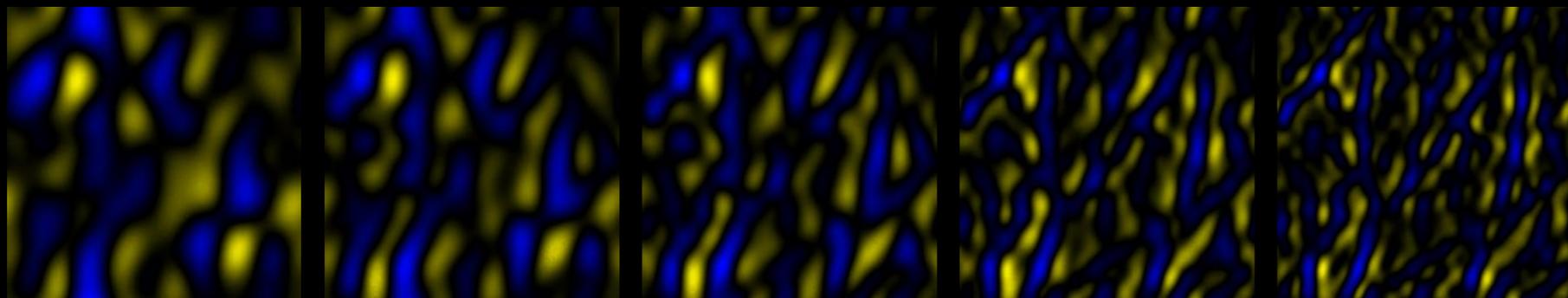
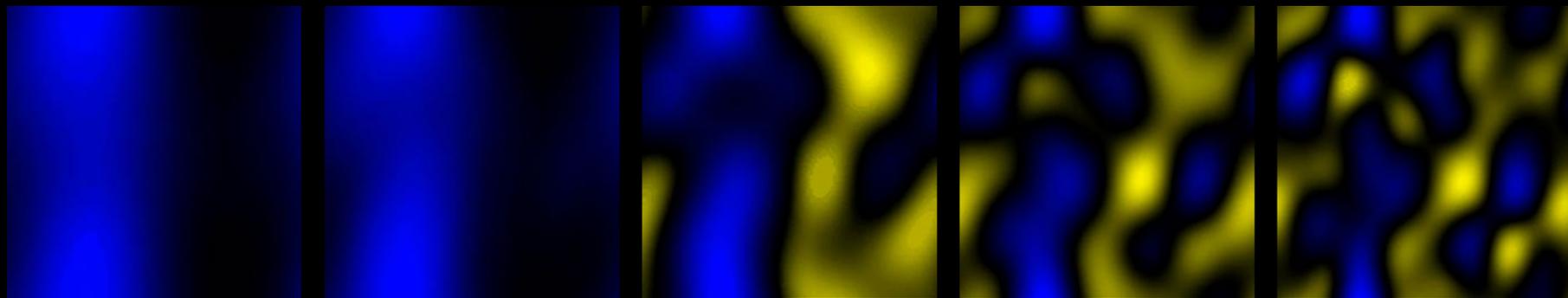
VIDEO

VIDEO

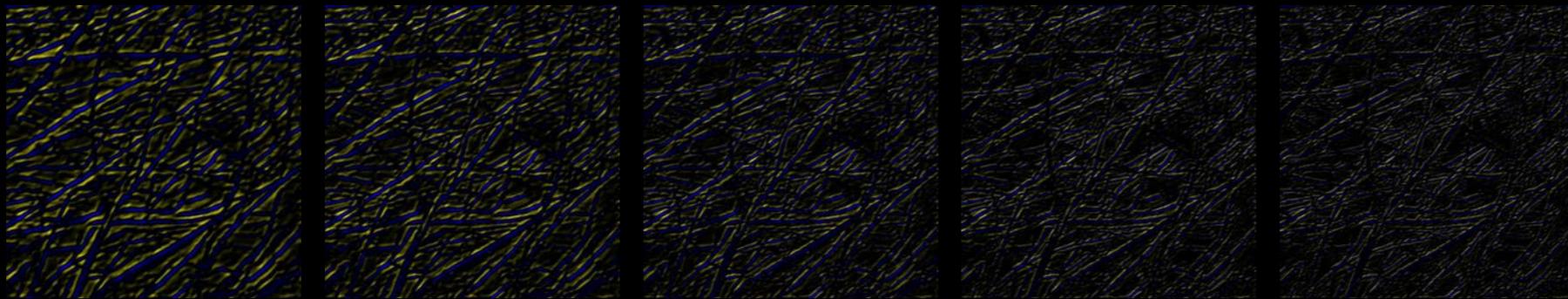
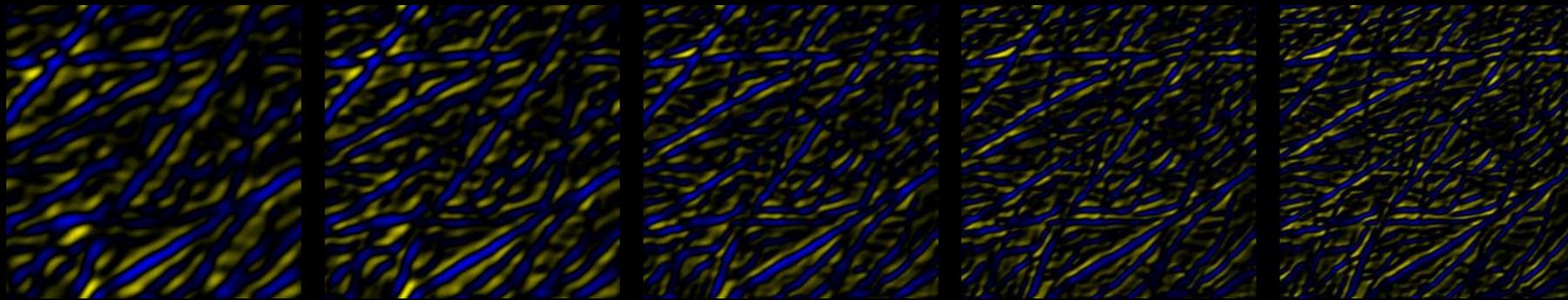
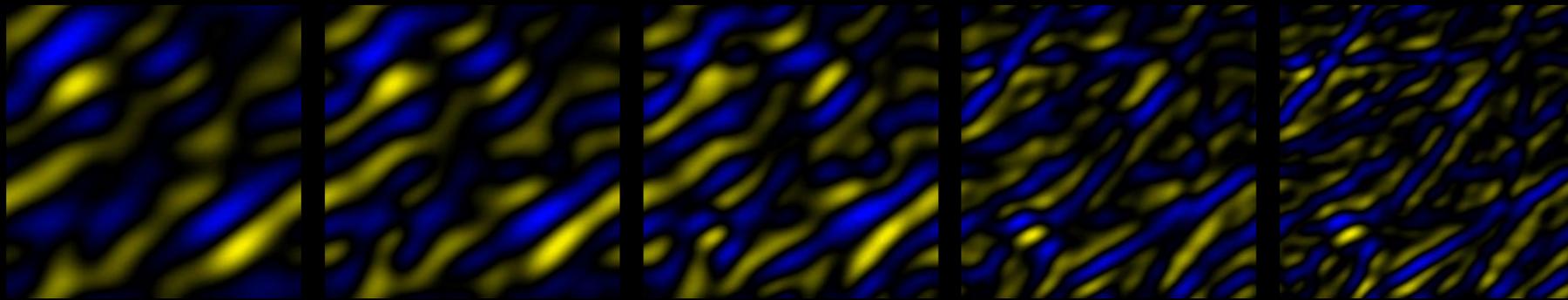
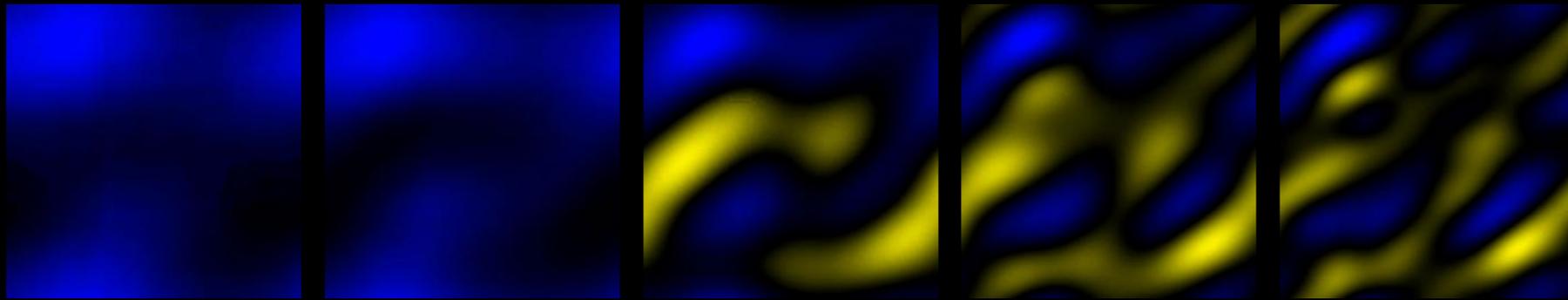
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for liness
there are at
least 3
components

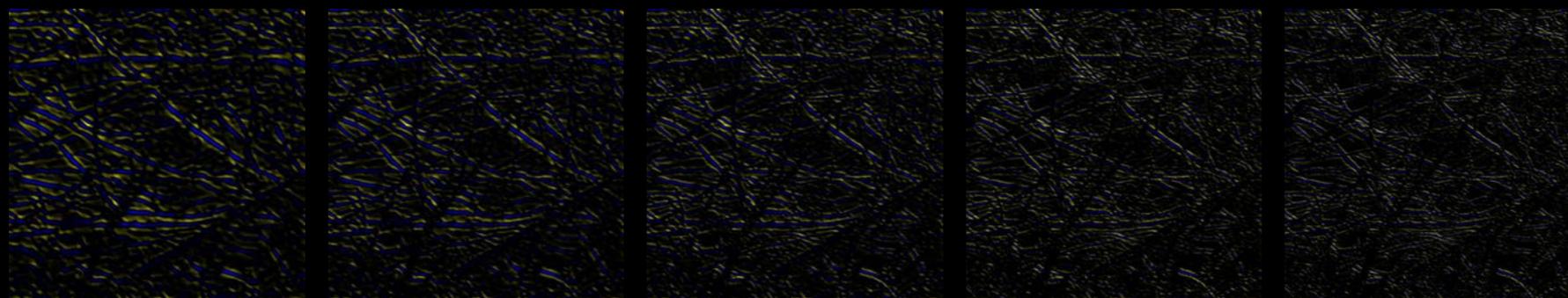
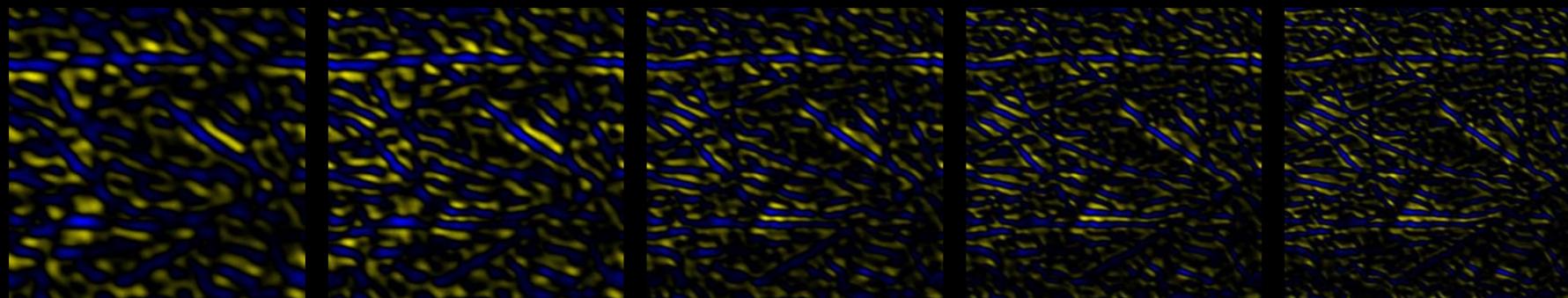
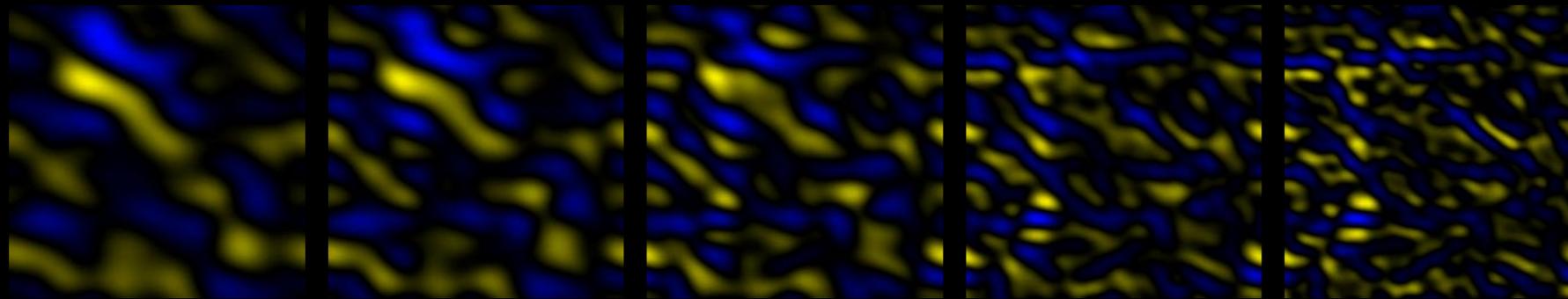
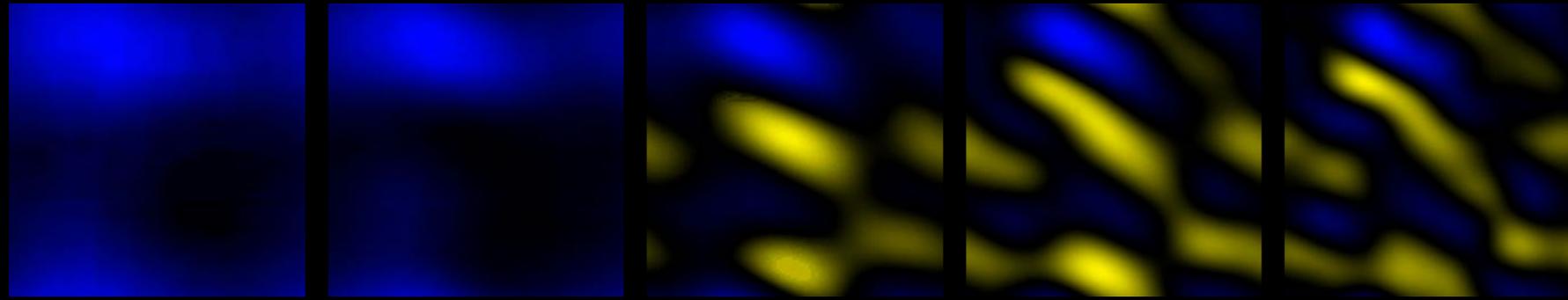




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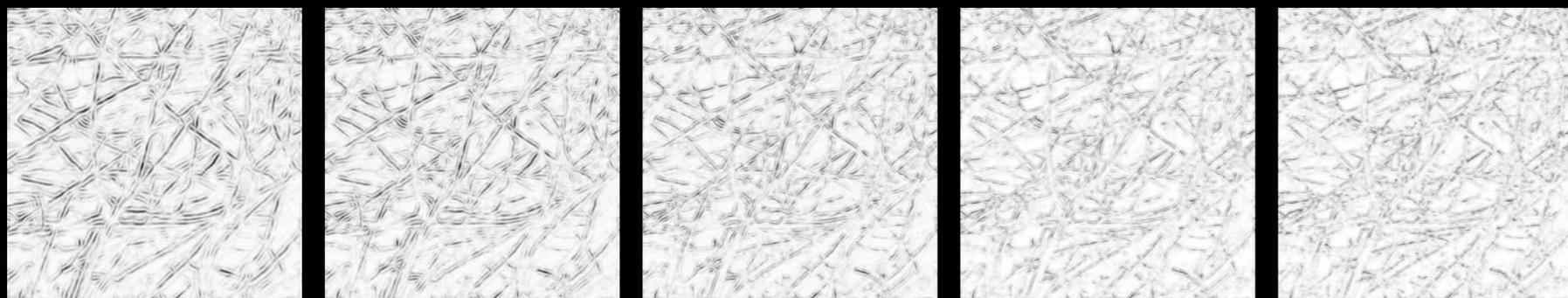
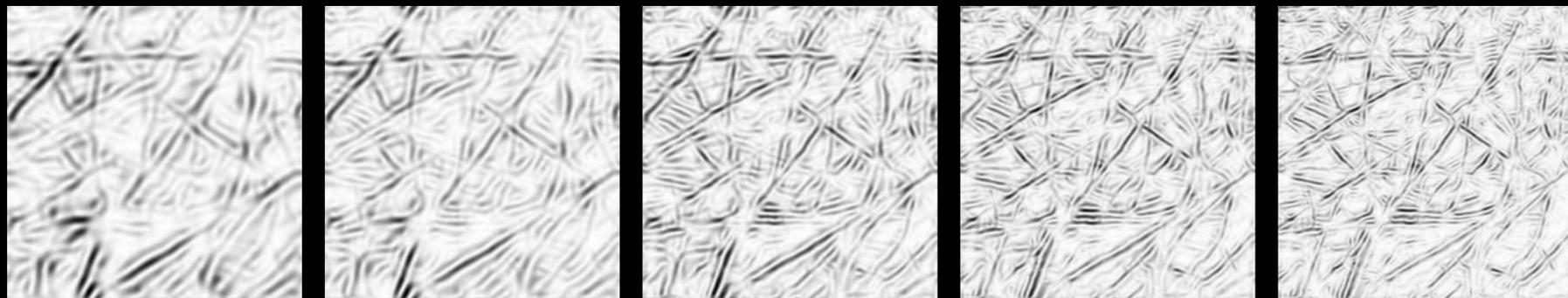
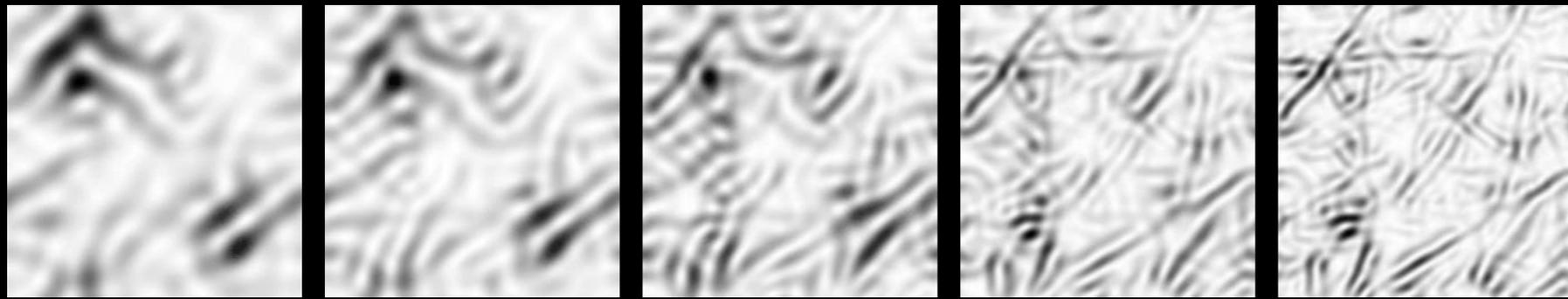


mosaic
of
video III

VIDEO

the components combined in
an overall lineness intensity





mosaic
of
video

the curvature of the lines of constant functional value $F(x,y)$ are proportional to

$$-\frac{\partial^2 F}{\partial x^2} \left(\frac{\partial F}{\partial y} \right)^2 + 2 \frac{\partial^2 F}{\partial x \partial y} \left(\frac{\partial F}{\partial x} \right) \left(\frac{\partial F}{\partial y} \right) - \frac{\partial^2 F}{\partial y^2} \left(\frac{\partial F}{\partial x} \right)^2$$

this is just an expression from differential geometry that you may find in math books

suppose the gradient is in the x-direction ($\partial F / \partial y = 0$) then you are left with

$$-\frac{\partial^2 F}{\partial y^2} \left(\frac{\partial F}{\partial x} \right)^2$$

that is a line detector signal times the square of an edge finder signal, $L_{yy} E_x^2$ say

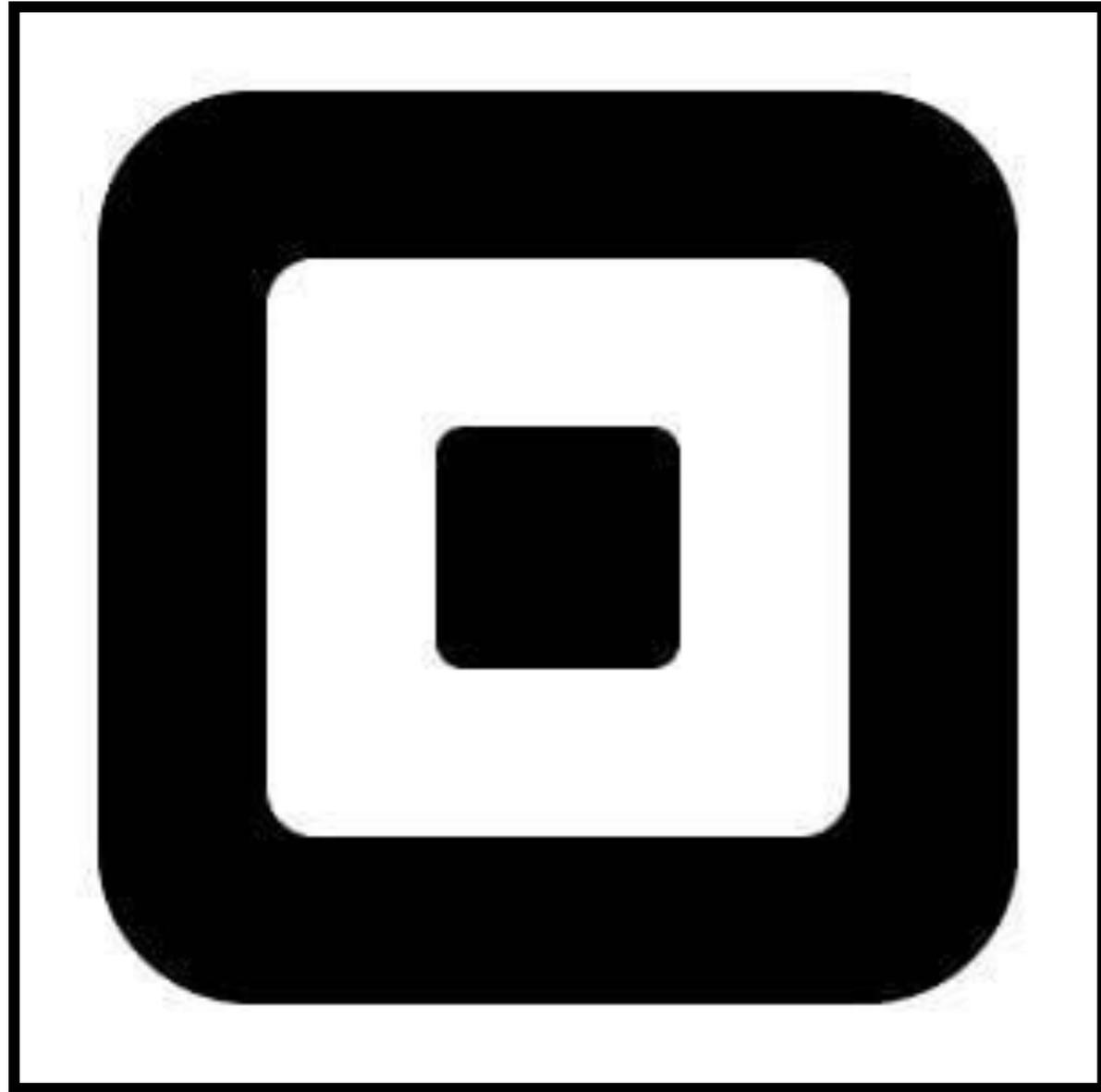
$$E_x \text{ squared } \times L_{yy} = \frac{\partial^2 F}{\partial y^2} \left(\frac{\partial F}{\partial x} \right)^2$$

this involves scaling, addition and multiplication operations that can be implemented at the synaptic level

doing this for ALL directions and averaging yields the result of the full expression $-\frac{\partial^2 F}{\partial x^2} \left(\frac{\partial F}{\partial y} \right)^2 + 2 \frac{\partial^2 F}{\partial x \partial y} \left(\frac{\partial F}{\partial x} \right) \left(\frac{\partial F}{\partial y} \right) - \frac{\partial^2 F}{\partial y^2} \left(\frac{\partial F}{\partial x} \right)^2$
 the averaging makes it an operation between orientation columns

you use the RF's as a child's LEGO building blocks to turn formal expressions into neural machines!

this one is a "corner detector"

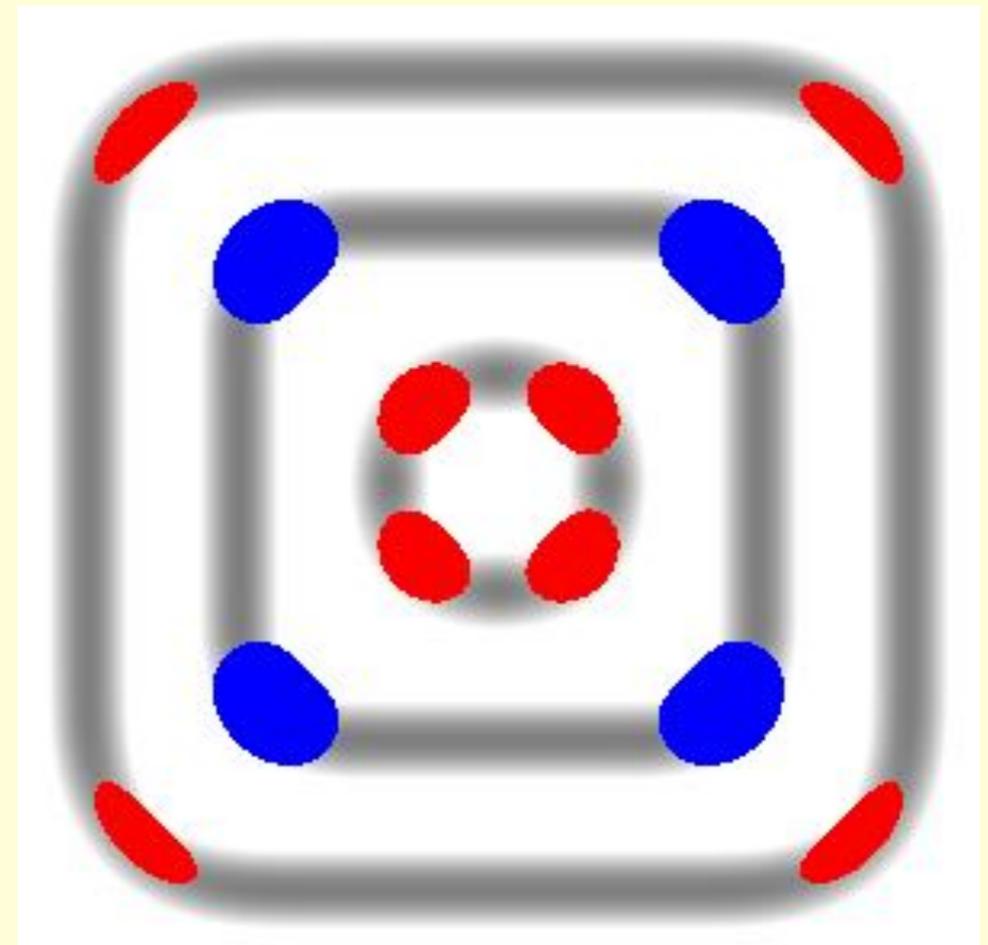


although constructed from "edge finders" and "line finders", the "corner detector" hardly responds to either edges or lines ("edge finder" and "line finder" are unfortunate terms)

notice that there are two kinds of corners!
here the corners are superimposed over the
edginess

the cornerness here disambiguates the
edginess

the 2nd order (line finder) edge
representation also disambiguates the
edginess (to be demonstrated later)



... nature is written in that great book which ever is before our eyes - I mean the **universe** - but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written ...

the book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; **without which one wanders in vain through a dark labyrinth ...** (Galileo, *Il Saggiatore*, 1623)

if there can be said to be something like a LINGUA FRANCA of the brain, i feel it has to be local, but multiscale, differential geometry

but i really feel that the idea of a language in terms of a two-sided communication is mistaken

more likely, the notion of "messaging" applies, like in object oriented algorithmic languages

in the case of messaging the "communication" is one-sided and the two "communicating" objects need not share any common understanding at all

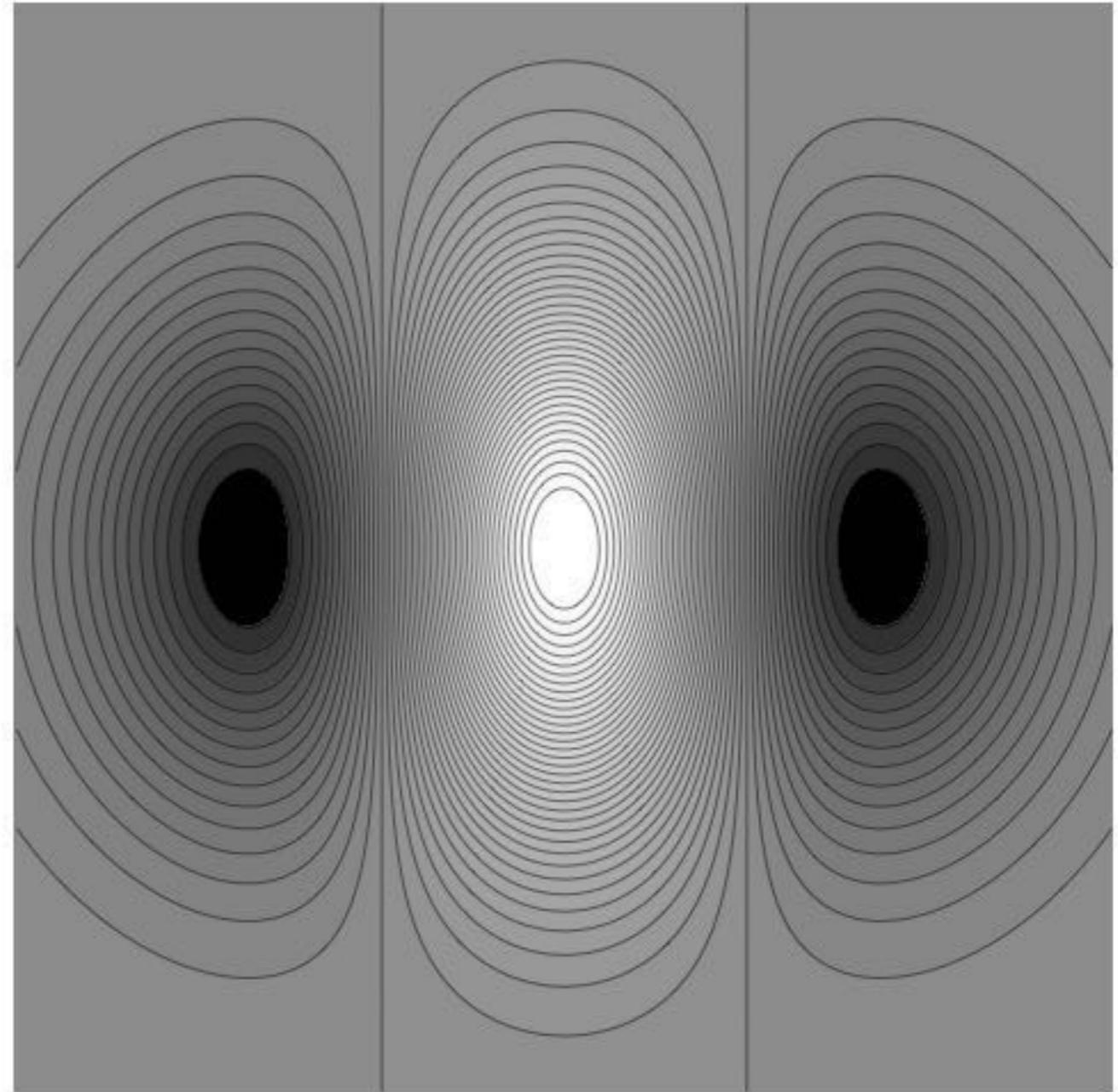
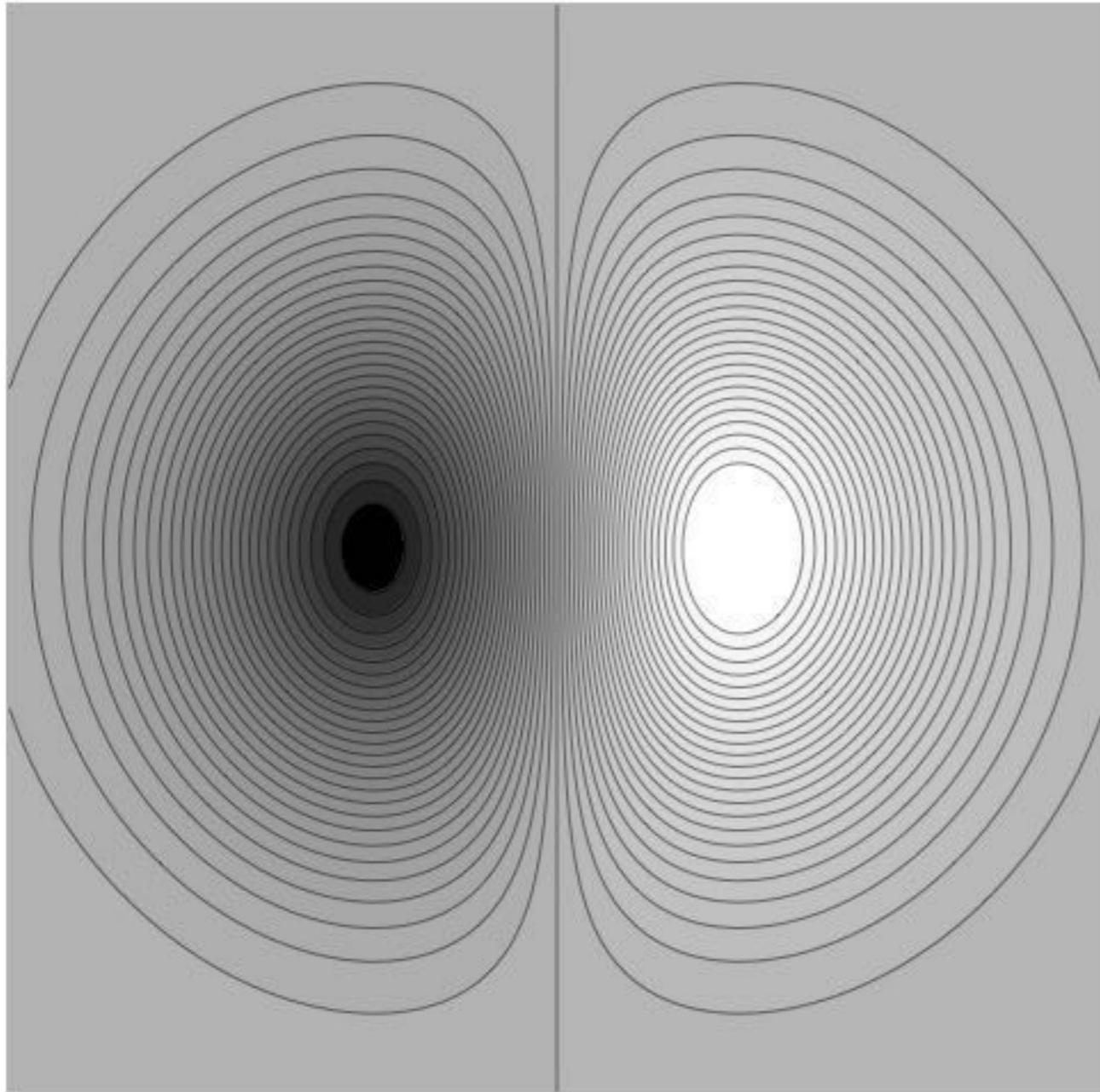
for instance, switching on the light in a room is an example of messaging, the message being mechanical (a button press say) in this case - on reception of the message the house's electrical systems powers the light bulb

while the simplest perceptive fields are well described in terms of Brentano's PLEROSIS, an object such as the "corner detector" cannot be captured that simply

the natural language to describe cortical objects in terms of function is differential geometry

unfortunately, neurophysiology is not yet ready to acknowledge (or better: exploit!) this

of course, "edge" and "line detectors" are perhaps to be understood as Kindergarten level differential geometry



the 1st and 2nd-order directional derivatives are just the simplest instances of *ATOMIC* multi-point operators (in brain science "atomicity" is implied by a single axon output)



edginess "represents" how much edge there is at some location - it does not LOOK like an edge

an "edgelet" PF looks just like an "edge detector" RF

the edgelet is like the "touch" of a painter, the strength of the touch is taken proportional with the edginess

thus the 1st-order is applied TWICE over, once to represent the edginess, then to present the edgelet

the combination has a 2nd-order profile

notice that

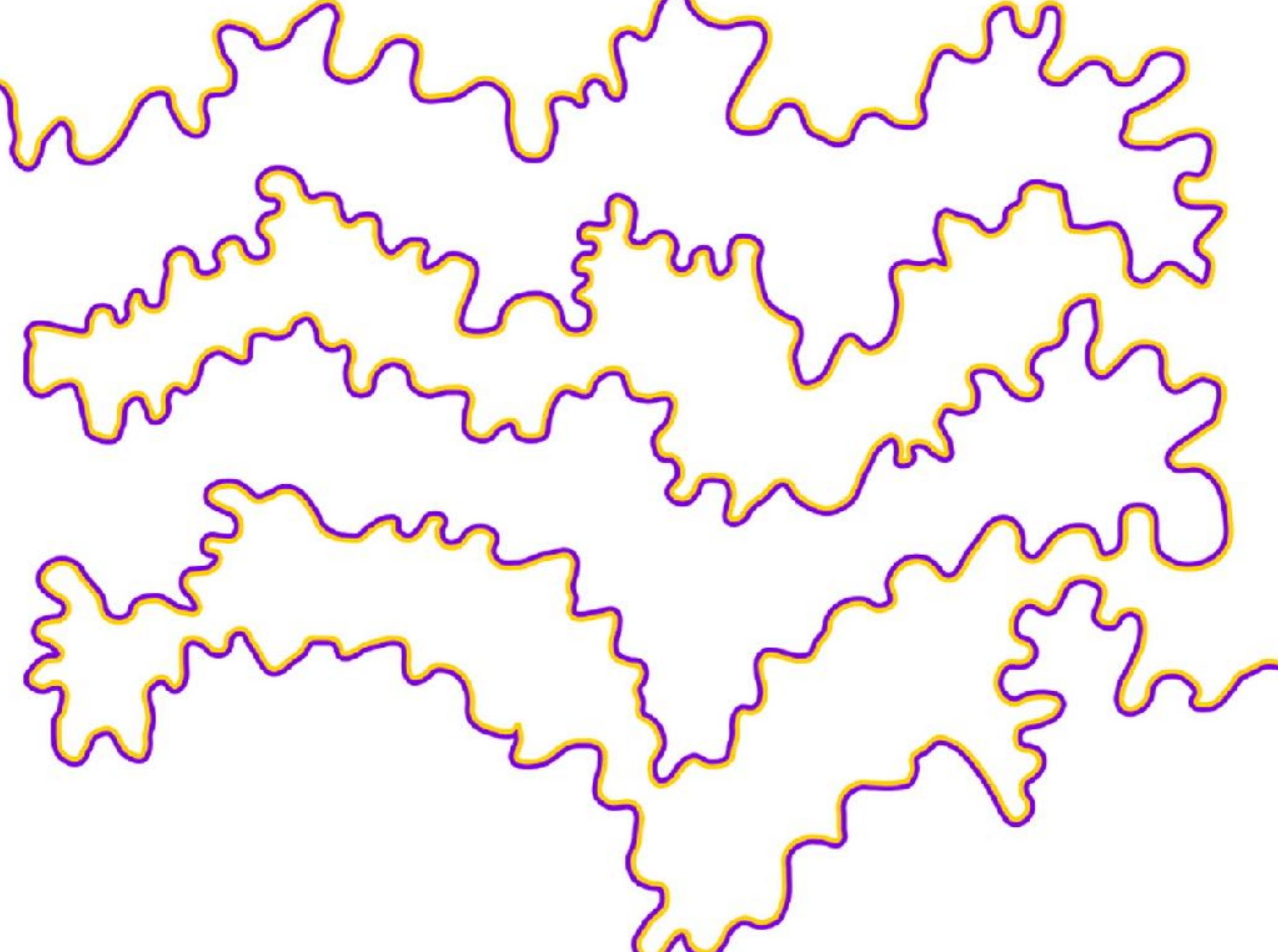
$$\frac{\partial}{\partial x} \left(\frac{e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \right) \circ \frac{\partial}{\partial x} \left(\frac{e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \right) = \left(\frac{e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \right) \circ \frac{\partial^2}{\partial x^2} \left(\frac{e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \right)$$

where the funny circle denotes concatenation, that means algebraically a convolution



here the
"edgelets"
PRESENT
the edges

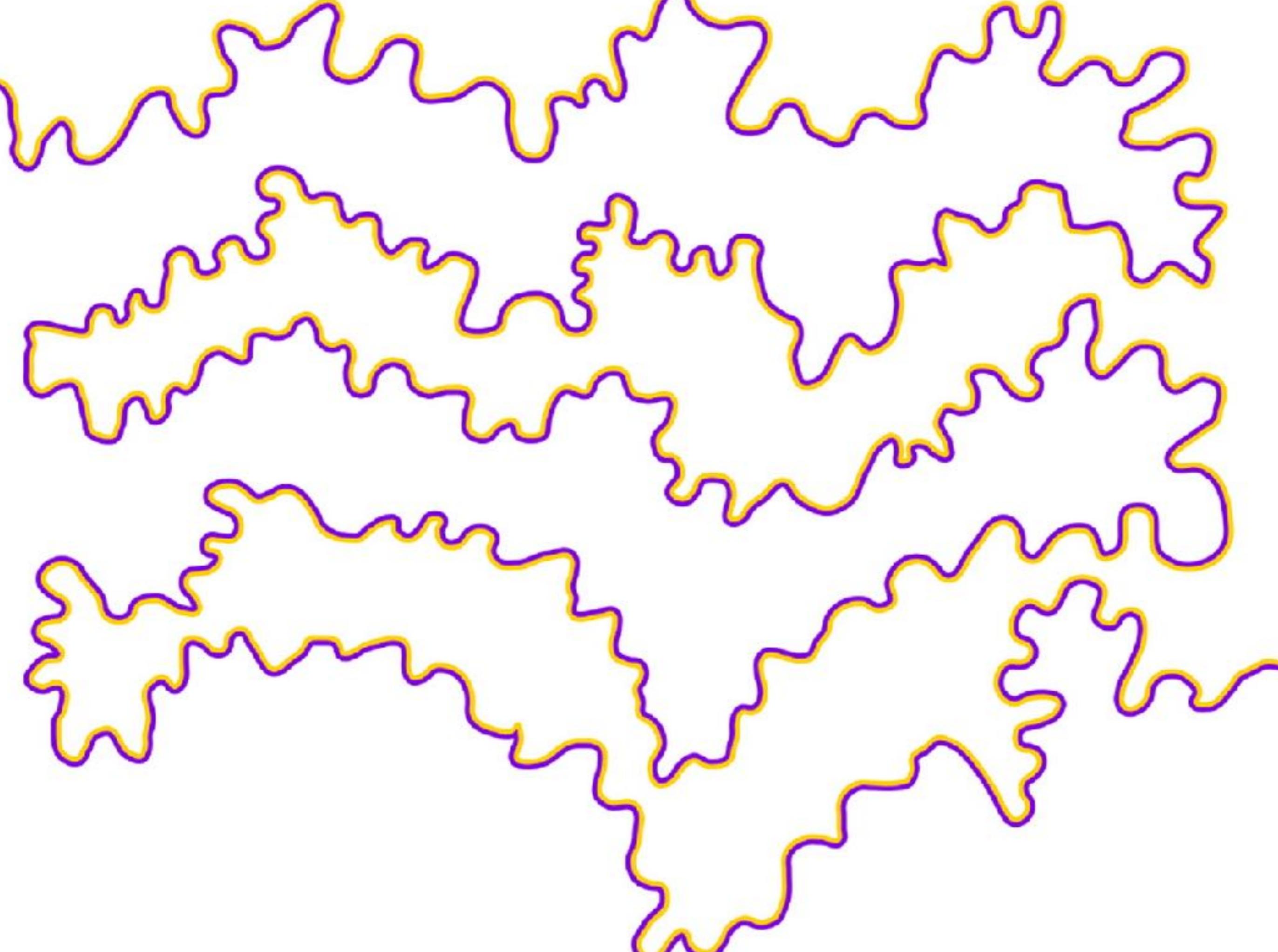
this is an
instance of
PERCEPTIVE
as distinct
from
RECEPTIVE
fields





Baingio Pinna's

"watercolor illusion"





the representation by RF's is the **end-stage of the front-end visual system** (beyond here neurophysiology loses track)

it is complete, objective and meaningless

the presentation by PF's is visual awareness, the **end-stage of psychogenesis**

it is selective, subjective and meaningful
(here psychology is clueless)

the representation by RF's is the end-stage of the front-end visual system (beyond here neurophysiology loses track)

it is complete, objective and meaningless

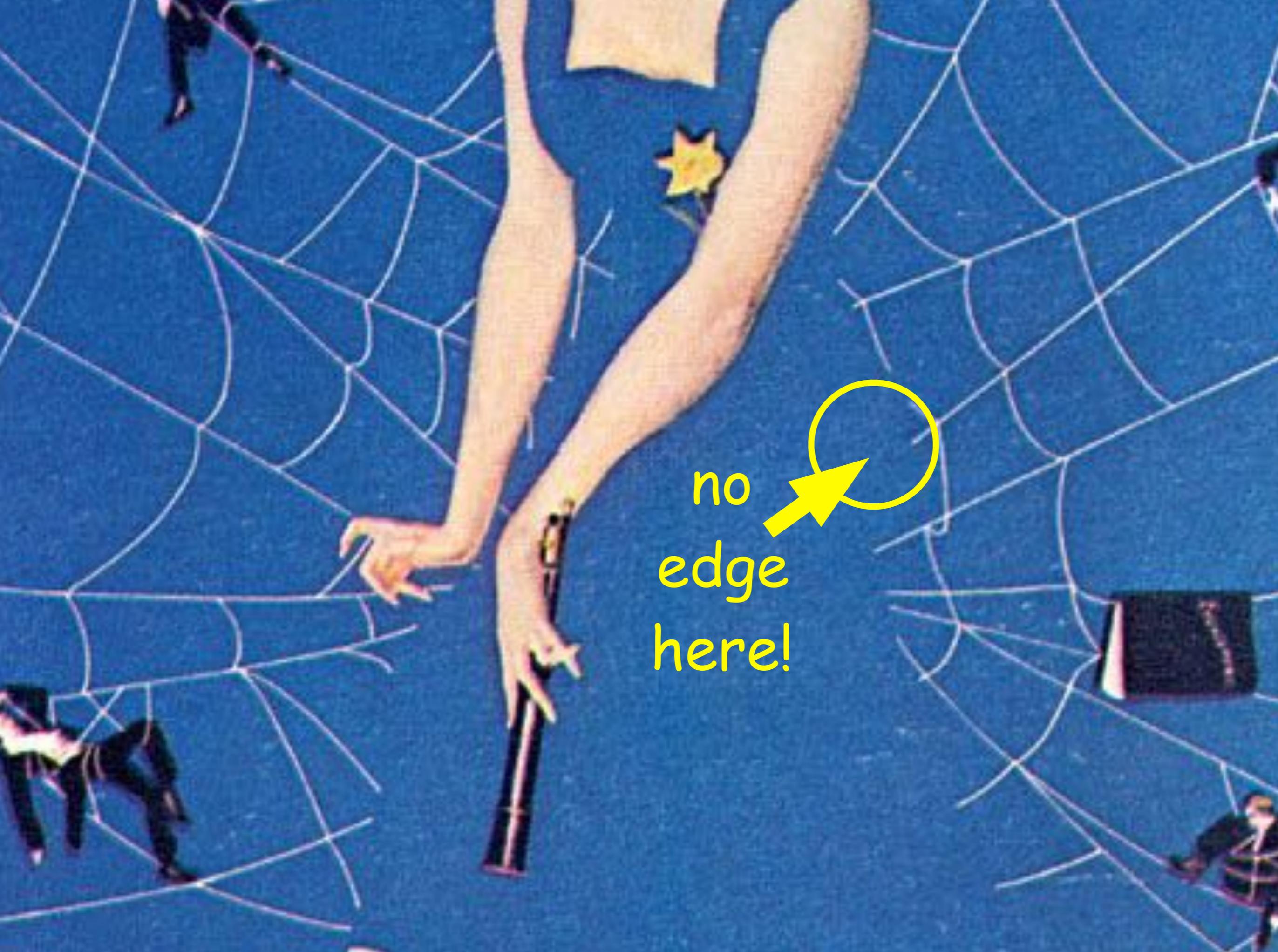
the presentation by PF's is visual awareness, the end-stage of psychogenesis

it is selective, subjective and meaningful (here psychology is clueless)



all perception
is necessarily
creative

one does not
need any edge
detector in order
to become aware
of an "edge"!



no
edge
here!

the vanishing girls where the speciality of Clarence Coles Phillips (1880-1927), an American artist and illustrator



change of gears:

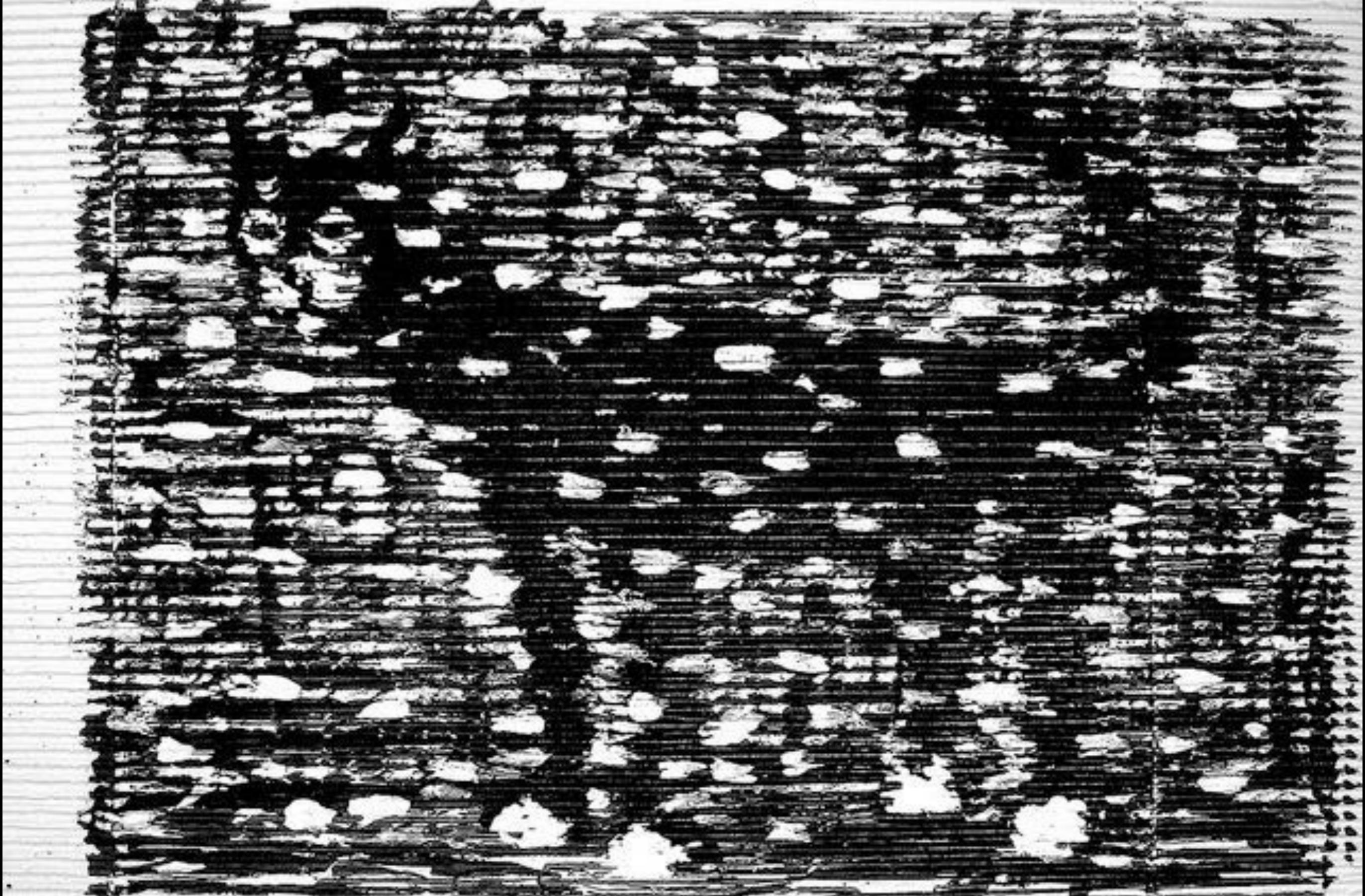
I'm going to talk about the other side
of the mirror

neural representation is one thing,
visual awareness (psychogenesis) is
another

change of gears:

I'm going to talk about the other side
of the mirror

neural representation is one thing,
visual awareness (psychogenesis) is
another

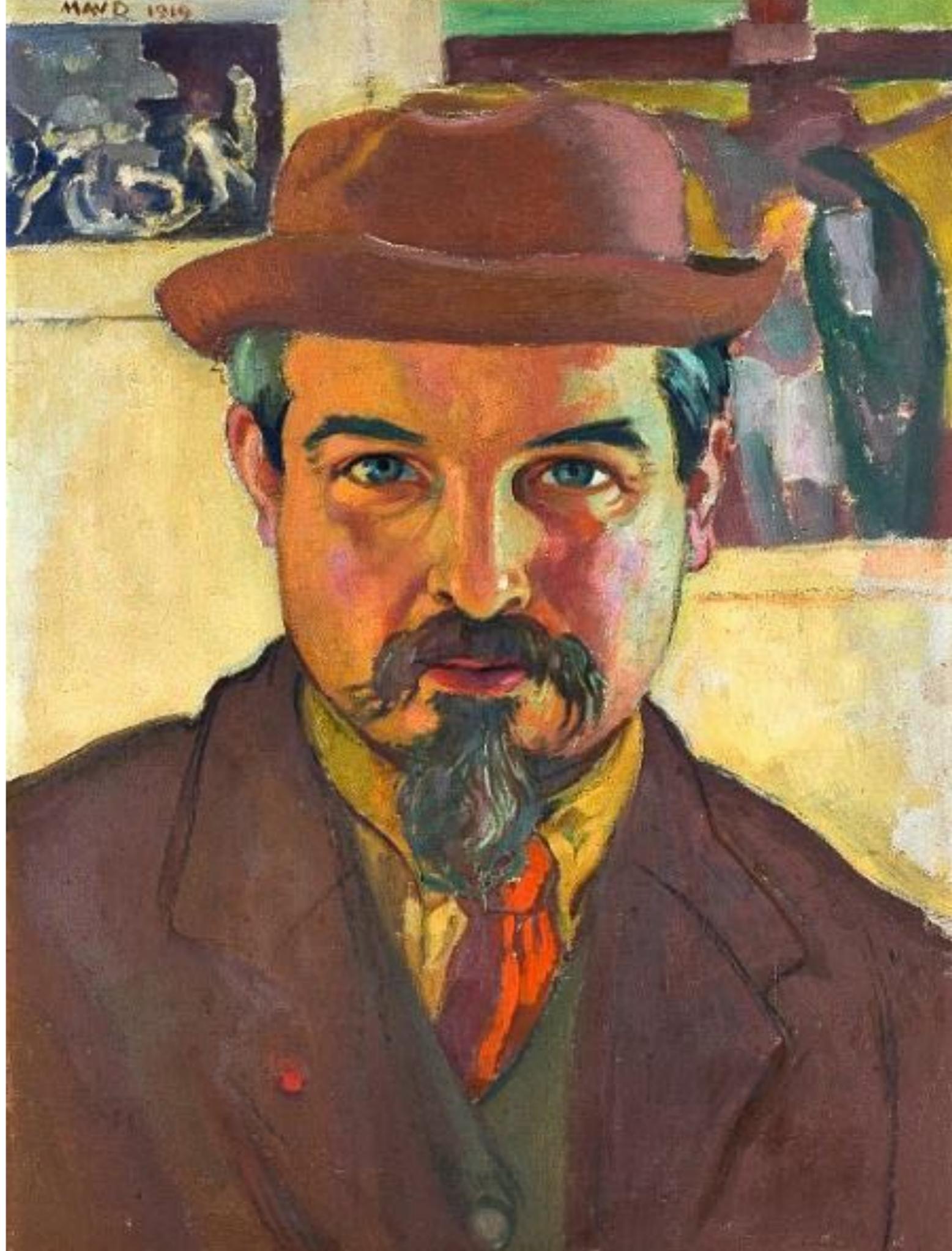


parts of awareness are evidently "hallucinated"

Franco Matticchio - *Gatto delle Nevi*

remember that a painting, before being a battle horse, a nude woman, or some anecdote - is essentially a flat surface covered with colors assembled in a certain order

Maurice Denis,
manifesto 1890





a wonderful painting by Maurice Denis

is it more than "a flat surface covered with colors assembled in a certain order"?

if so, what is the MORE?

to the behaviourists speech was "the movement of air molecules"

this is no doubt a deep truth - do you consider it the "whole story"?

if so, you are a TRUE SCIENTIST!

people who cherish the "more" have some trust in PHENOMENOLOGY, which is not at all a science, because by definition SUBJECTIVE

i wouldn't say they are mad, they simply feel that science doesn't exhaust what there is to understand

SCIENCE involves a self-inflicted tunnel-vision that is often very useful, but in other cases it may be useful to go beyond it

the paint on the canvas is like the neural activity in the brain, **physical structures** arranged in a certain way

the colors in visual awareness are **mental elements** in some meaningful pattern

how do qualities and meanings come about? - the **problem of psychogenesis**



Erwin Schrödinger

1887

- 1961

a spark of awareness
occurs when an
expectation fails to
fit the poking of the
physical environment

Schrödinger frankly offered a PROPOSITION since he knew very well that he was operating at the very edge of science

a proposition has purely heuristic meaning, it cannot be proven true or false by science, not now, nor in the future

so it is scientifically MEANINGLESS

yet some propositions have considerable HEURISTIC value
at least to me, Schrödinger's proposition has

notice the difference to such notions that there is some centre of consciousness in the brain

- taken as a proposition it has as much heuristic value as the belief in angels
- taken as a scientific hypothesis it makes no sense since consciousness does not exist (scientifically)
- taken as a scientific program it serves to send people on irrelevant (there is no goal) ghost hunts

i'd rather take Schrödinger's proposition and see where it gets me

psychogenesis proceeds
by active probing

it explores likely plots
by hunting for clues



questions implicate
possible answers, they
turn mere structure
into meaning

this is what renders
presentations
intentional



meaning is due to
the fact that visual
presentations are
constructions

meaning cannot be
computed



psychogenesis
constructs through
prolific diversification
and merciless pruning

not the assemblage of
parts



awareness is necessarily "perspectival"

(patchwork,
incomplete)

衆瞽
摸象之圖



Itcho Hanabusa

DO WE "SEE" HUMAN PRESENCE?



"human presence" is detected if you spot...

- a human
- a human foot
- a depression possibly due to a human foot
- retinal activities due to depressions of human feet
- certain brain activities, due to...

virtually anything is detected via multiple indirection!

brain activity is nothing special in this series

what is decisive is intention, probing, questioning
the world - (the brain is part of the world)

depressions in the sand are meaningless structures,
is it any different with cortical activities?

cortical activity is a proxy for the world

in this view the "representation" of the cortex is not essentially different from the "representation" of sand of the beach

thus the cortex does in no sense "compute awarenesss"

awareness is due to a process that may be called PSYCHOGENESIS

psychogenesis is controlled hallucination, the "control" involves checking the cortical representation for clues and evidence

psychogenesis is much like Sherlock Holmes, a forensic investigator, that **uses** the cortical representation as a forensic file

this implies that the bulk of the cortical representation will be ignored

psychogenesis selects that what is useful to make a case "beyond reasonable doubt"

back to the points:

where are they?

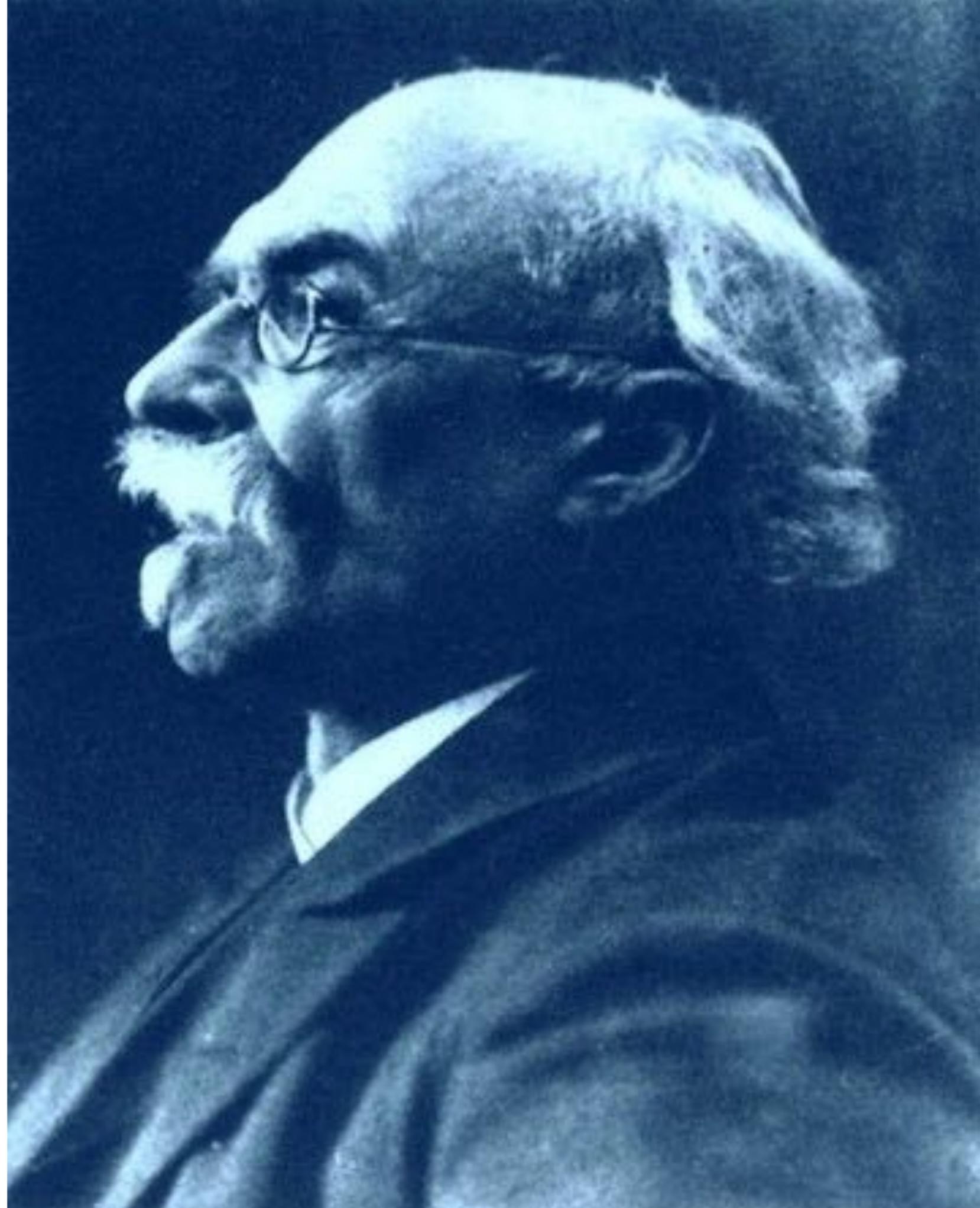
Rudolf Hermann Lotze

1817 Bautzen

- 1881 Berlin

a qualitative property ... which the impression acquires ... in virtue of the peculiar nature of the place at which it comes into contact with the body ...

... act as marks or as **local signs**, under whose guidance it proceeds in spreading out the impressions into an image occupying space ...



Hermann Lotze's "Localzeichen" was taken very seriously by scientists like Helmholtz

nowadays few people have ever heard of the term, yet the problem of local sign was never resolved

mainstream science has decided to ignore it

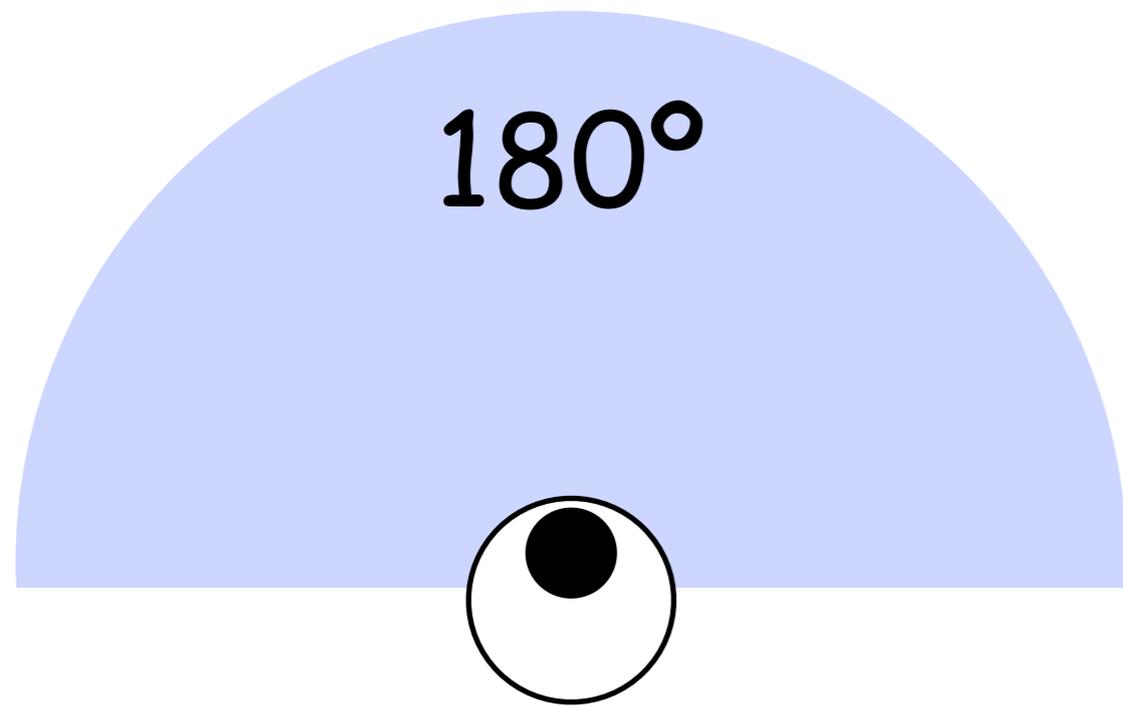
i have little doubt that the problem will eventually emerge again, but since it relates to psychogenesis, it cannot be dealt with in science per se, it will require a form of psychology that can live with phenomenology

as long as psychology deludes itself to be mere "dry physiology" the problem of local sign will necessarily remain submerged

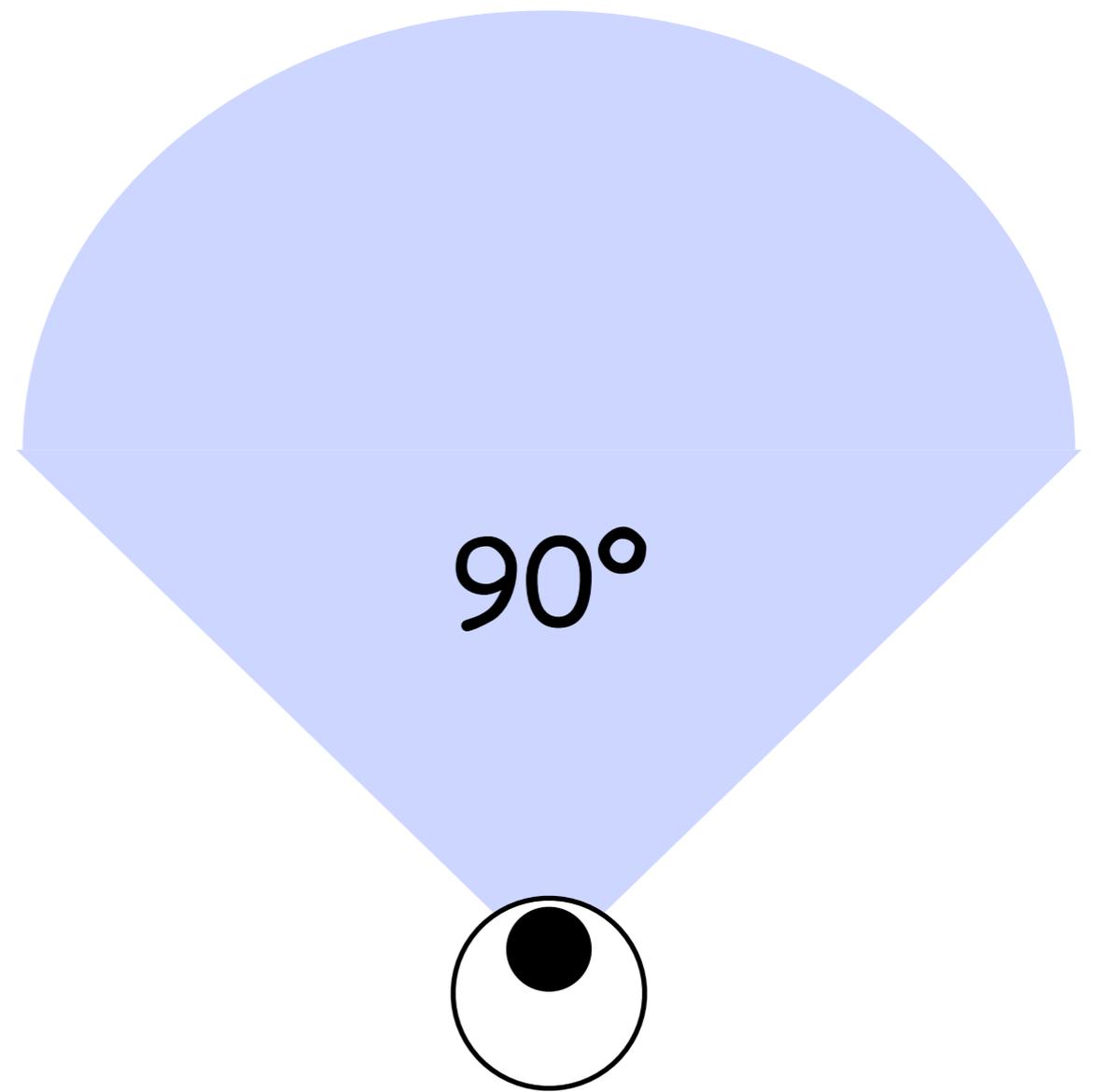
- the "field of view" is an object of physics
- the "visual field" is a mental object
- Lotze's "local sign" is a **QUALE** that correlates roughly with physical **LOCATION**

such correlations are neither necessary nor precise

external local sign



the visual field of
the human eye
subtends about a
halfspace



but most people feel
that their visual
field subtends
"about a right angle"

in his monumental "Physiologische Optik" Helmholtz explains the width of the optical field of view of the eye and then notices that - much to his surprise - the extent of his visual field APPEARS to him as "much closer to a right angle"

apparently, he was like the majority of people in this respect (including me)

yet, almost nobody spontaneously notices this!

one other (rather singular and surprising) example is Kepler (Astronomiae Pars Optica, 1604) who must have been a remarkable observer

the ancient Greeks reckoned the visual field to subtend a right angled cone, apparently they shared the same illusion (but, of course, they were looking for some ideal here!)

the commonly found fact that the Euclidean "visual rays" are parallel in your awareness instead of fanning out from the eye leads to a variety of misjudgments in visual space

we find that errors in the judgement of spatial attitudes of objects in your ken may well amount to up to a hundred degrees!

perhaps remarkably, such enormous errors in spatial judgements have not been recorded in the standard textbooks on perception, whereas otherwise so much attention is placed on minor (say minutes of arc) deviations

mainstream vision research is very selective and simply ignores all that fails to fit its current convictions

"strabismic amblyopes" have good visual acuity and contrast sensitivity

yet the strabismic eye fails to yield detail for the visual field is "scrambled"

the points are not at their correct places

here **internal local sign** is disturbed

the paper by Robert Hess flabbergasted me on first reading: here were people with PERFECT visual acuity and PERFECT contrast sensitivity, great visual systems (at least, by (still) current scientific standards) who nevertheless didn't manage to SEE much!

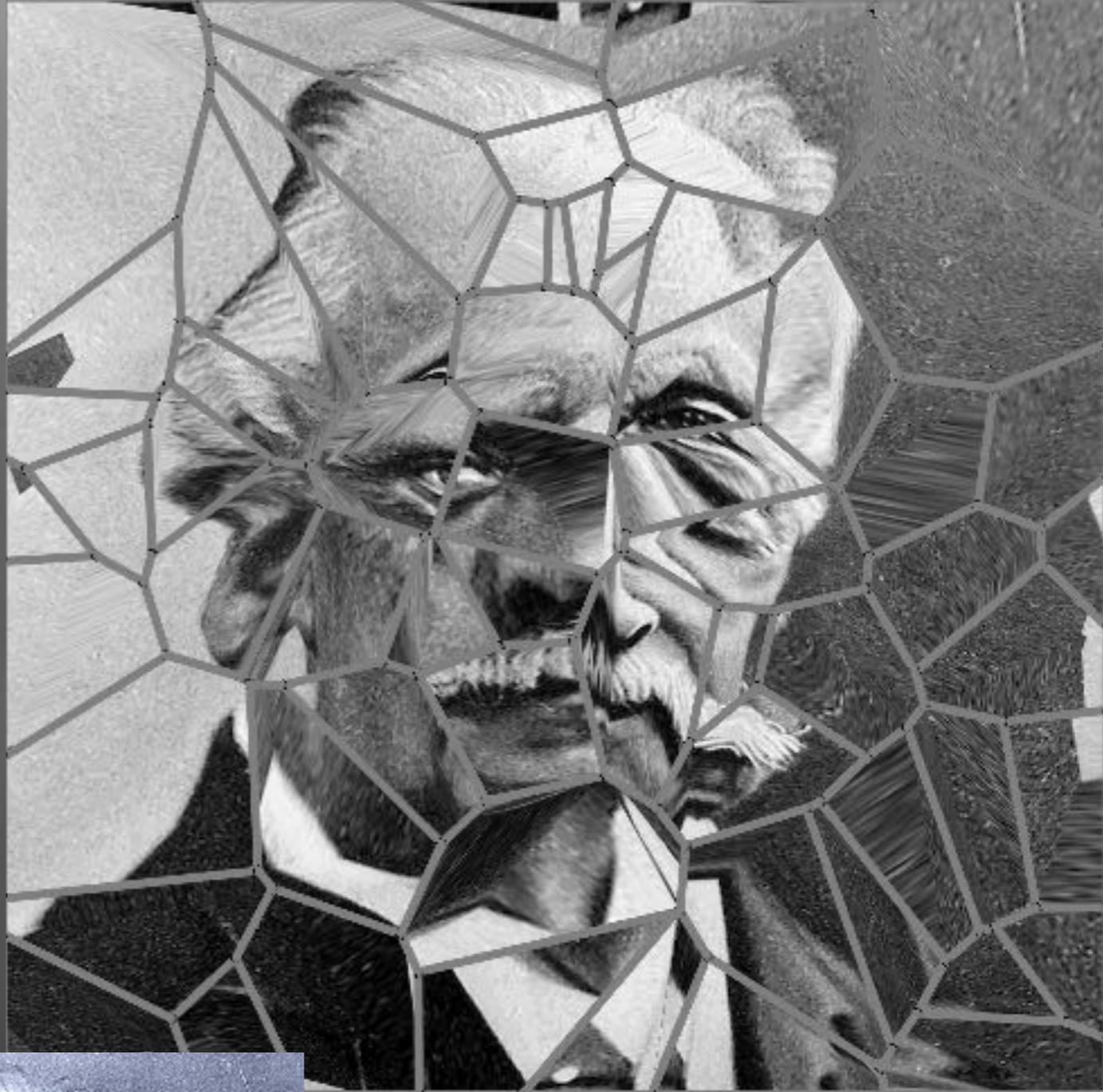
Hess had the guts to ask these people WHAT they saw - scientifically speaking, that is strictly NOT DONE, indeed BAD PRACTICE, he should be ashamed as a scientist

yet (to me) the response of his observers was very enlightening: they clearly saw all the PIECES but were unable to make sense of the WHOLE: here was a perfect case of defective Lotze Localzeichen!

hence the term "tarachopia": "scrambled visual field"

this proves (to me at least) that Lotze's local sign is not a mere philosophical fiction, but that it is very REAL

at least when you believe these poor observers, for after all, they were merely emitting disturbances in the movements of air molecules - yet their vision was unexplainable substandard, an objective fact to be accounted for



myopia

spectacles help



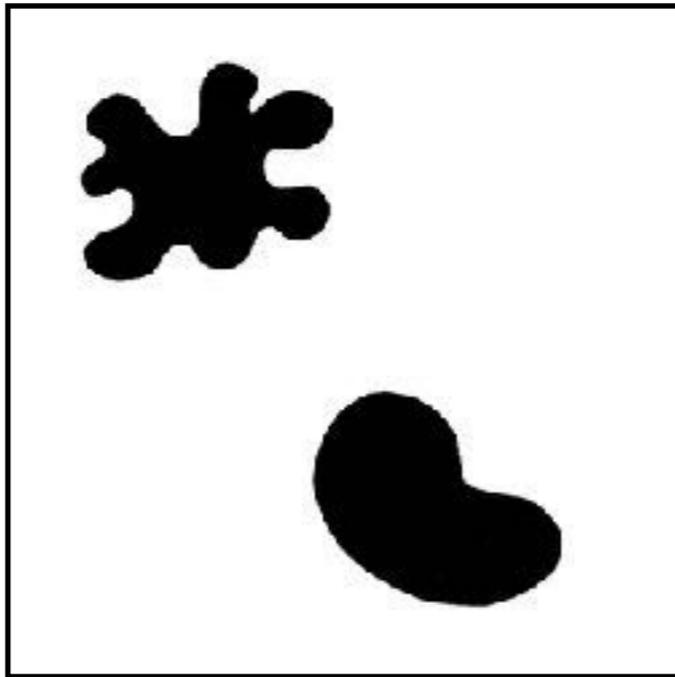
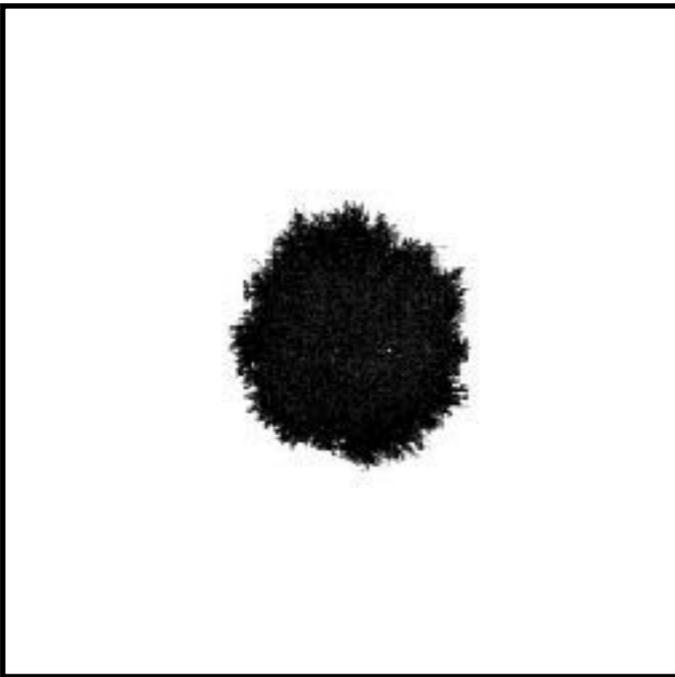
tarachopic
amblyopia

an agnosia

of course, it is not possible to ILLUSTRATE tarachopia, the image at right is just a schematic illustration of visual field scrambling

notice that the effects of myopia (image at left) are optical and thus can be corrected by suitable spectacles

there are no spectacles that will cure tarachopia, it is not of physical (that is optical) origin, it is a "soul blindness" (Seelenblindheit), that is a deficient psychogenesis



an easy task:

how many (one or two?) black blobs are there in each box?



a hard task:

how many (one or two?) black blobs are there in each box?

we are all tarachopic amblyopes to some extent

to some extent we are all tarachopic due to the fact that we fail to deal with even mild complexity

i never met anyone who solved the matter illustrated at bottom *AT A GLANCE*, which is what most people are able to do in the case illustrated at top

(bottom example due to Minsky and Papert)

OMITTED MATERIAL

at this place I had a number of videos illustrating the effect of scrambling the optical input

this demonstrates that visual awareness can be coherent even in cases where the optical structure available to the observer is not!

it implies that vision is not veridical, but a variety of controlled hallucination - local sign is MADE and adjusted as needed for vision

such examples have been known for a century (Benussi's work on sounds at the Graz laboratory at the early 20th c. perhaps first), but are conveniently ignored by mainstream perceptual science

sorry that I can't include the videos in this simple PDF, but you can watch them in another de Cloutcrans Press booklet "ChronoGeometry", which you can freely download

so people are not good at spatial tasks
(or maybe too good?)

apparently **points do not possess an absolute location** (as receptive fields do)

they are created on the fly in psychogenesis
and are rather less tangible than is often
supposed

there cannot be a fixed relation between

the **receptive** fields of **neurophysiology**

&

the **perceptive** fields of **phenomenology**

The background is a painting of a rural landscape. In the foreground, several large haystacks are visible, rendered with a dense, textured style using warm colors like yellow, orange, and red. In the middle ground, there is a small, simple building with a dark roof, possibly a barn or a farmhouse. The sky is a pale, hazy blue with some light, wispy clouds. The overall style is impressionistic, with visible brushstrokes and a focus on light and color.

what & where are the points?
... a never-ending story ...

Charles Angrand (1854-1926), *The Harvest* (1890)

the pointillist painter puts points on the canvas

but is the pointillist blue sky thus "composed of points"? DEFINITELY NOT!

these points are on the CANVAS whereas the blue sky is in AWARENESS
(remember Maurice Denis' dictum!)

when you see the blue sky is a uniform, (point-)textured area, the points have no individual existence, you could do as well by hatching using lines or scribbles

this is indeed a nice illustration of Maurice Denis' distinction

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ABOUT THE CLOOTCRANS PRESS

The Cloutcrans Press is a [selfpublishing](#) initiative of Jan Koenderink. Notice that the publisher takes no responsibility for the contents, except that he gave it an honest try—as he always does. Since the books are free you should have no reason to complain.

THE “CLOOTCRANS” appears on the front page of [Simon Stevin’s](#) ([Brugge](#), 1548–1620, [Den Haag](#)) *De Beghinselen der Weeghconst*, published 1586 at [Christoffel Plantijn’s](#) Press at [Leyden](#) in one volume with *De Weeghdaet*, *De Beghinselen des Waterwichts*, and a *Anhang*. In 1605 there appeared a supplement *Byvough der Weeghconst* in the *Wisconstige Gedachtenissen*. The text reads “*Wonder en is gheen wonder*”. The figure gives an intuitive

“eye measure” proof of the [parallelogram of forces](#).

The key argument is

de cloten sullen uyt haer selven een eeuwich roersel maken, t’welck valsch is.

Simon Stevin was a Dutch genius, not only a mathematician, but also an engineer with remarkable horse sense. I consider his “cloutcrans bewijs” one of the jewels of [sixteenth century](#) science. It is “[natural philosophy](#)” at its best.

